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STATISTICAL MODEL CALCULATIONS
FOR PHOTO NUCLEAR REACTIONS

MONTECU J. LOWRY

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STATISTICAL MODEL CALCULATIONS
FOR
PHOTO NUCLEAR REACTIONS

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MONTECUE J. LOWRY

STATISTICAL MODEL CALCULATIONS
FOR
PHOTO NUCLEAR REACTIONS
by

Montecue J. Lowry
11
Major, United States Army

Submitted in partial fulfillment of
the requirement for the degree of

MASTER OF SCIENCE
IN
PHYSICS

United States Naval Postgraduate School
Monterey, California

1965

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STATISTICAL MODEL CALCULATIONS

FOR

PHOTO NUCLEAR REACTIONS

by

Montecue J. Lowry

This work is accepted as fulfilling
the thesis requirements for the degree of

MASTER OF SCIENCE

IN

PHYSICS

from the

United States Naval Postgraduate School

ABSTRACT

The statistical model assumes a statistical distribution of nuclear density levels in the decay of the compound nucleus. The densities of nuclear levels can be described by a statistical formula in a region containing many energy levels. This formula can be used to predict the cross-sections for photonuclear reactions within the energy range 0 to 30 MEV. Provided assumptions are made neglecting the coulomb effect, spin, parity, angular momentum, and tertiary and higher order reactions, these predictions can be made by a digital computer and the results plotted on a graph with experimental data for comparison. The digital computer program is used to find the statistical level density formula that most closely approximates the cross-sections obtained experimentally. The formulas used thus far do not approximate the experimental data closely enough. Some formulas for the density functions give a shape for the cross-section as a function of energy that corresponds well with the experimental curve, but the ordinates are too small. The formula $P(E) = C E \exp(A E^n)$ is the best one found so far.

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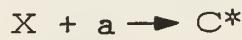
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1. Introduction

A description of a nuclear reaction in terms of the decay of long-lived equilibrium systems which neglect phase relations can be called a "statistical model". It is recognized that statistical theory qualitatively predicts the major portion of nuclear reaction cross sections and in this thesis a comparison is made between the statistical model and experiment. The reason for this is primarily that the decay of an equilibrium system is governed by the available phase space.¹ The use of nuclear models is an attempt to explain in tangible form the observed phenomena characteristic of nuclear reactions.

To explain large resonances in neutron cross-sections, Niels Bohr in 1936 introduced the concept of a compound nucleus which is a many-body system of strongly interacting particles formed by the combination of an incident particle a with a target nucleus:



In such a system the incident particle a has a short mean free path and shares its energy with the other particles of the system C^* so that it cannot be re-emitted until,

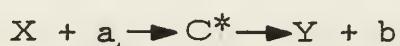
¹Ericson, Torleif, The Statistical Model and Nuclear Level Densities. Advances in Physics, v. 9, no. 36, Oct. 1960: 430.

as a result of further exchanges, sufficient energy can again be concentrated on this or on a similar particle. Should the incident particle be a slow neutron, this could be a length of time sufficient to permit the emission of radiation. The capture process is complete, and the compound nucleus C is formed in its ground state either by emission of a single photon



or by the emission of a sequence of quanta in cascade. This explains the predominance of the capture reaction and the suppression of elastic scattering.²

The progress of many types of nuclear reactions can be described in terms of the compound nucleus as a two-stage process:



in contrast to a single-stage process:



Niels Bohr observed the usefulness of dividing a nuclear reaction into two states:

- (1) the formation of the compound system
- (2) the disintegration of the compound system into the reaction products.

²Blatt, John M., and Weiskopf, Victor F., Theoretical Nuclear Physics. John Wiley and Sons, New York. 1952: 528-529.

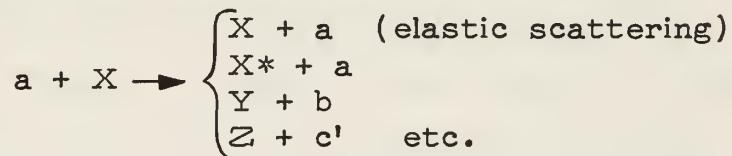
³Blatt and Weiskopf: 528-529.

These two states can be treated as independent processes in the sense that the mode of disintegration of the compound system depends only on its energy, angular momentum, and parity, but not on the specific way in which it was formed. This assumption has proved to be valid or approximately valid for many cases.⁴

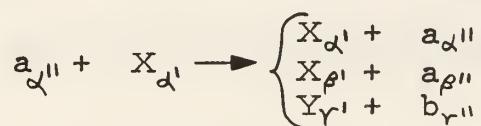
2. Theory of Nuclear Reactions

a. Channels

Consider the following nuclear reactions:



All of these reactions except that of elastic scattering can be subdivided according to the quantum state of the residual nucleus and the emerging particle. One can denote the states of these nuclei by α' , β' , γ' , ---, and the states of the incident or emerging particles by α'' , β'' , γ'' , ---. Should particles a, b, c, \dots , be elementary, states $\alpha'', \beta'', \gamma'', \dots$, refer to their spin orientation, and the following reactions result:



⁴Blatt and Weiskopf: 341.

α' and α'' are states of the target nucleus and the incident particle; thus, β' and β'' , or γ' and γ'' can denote any quantum state of X and a or Y and b respectively which can occur in this reaction. The conservation laws of energy, angular momentum, and parity restrict the possible combinations of β' and β'' , γ' and γ'' , etc. Any such possible combination of residual nucleus and emerging particle, each of which is in a definite quantum state, is called a reaction channel. These channels are denoted by single Greek letters α , β , γ , ---, which consist of both indices α' and α'' or β' and β'' . Channel $X_{\alpha'} + a_{\alpha''}$ is called the entrance channel or initiating channel of the above reaction.⁵

b. Reaction Probability

According to the Bohr assumption, one can write the cross-section of a nuclear reaction $X(a,b)Y$ in the form:

$$\sigma(a,b) = \sigma_c(a) G_c(b)$$

where $\sigma_c(a)$ is the cross-section for the formation of a compound system by particle a incident upon the target nucleus X. $G_c(b)$ is the probability that the compound

⁵Blatt and Weiskopf: 313.

system C, once formed, decays by emission of a particle b, leaving a residual nucleus Y. $G_c(b)$ is a pure number; the compound system C must eventually decay in some special way; $G_c(b)$ is the probability for this special mode of decay.⁶

It is useful to describe the reaction X(a,b)Y in more detail by considering the cross-section $\sigma(\alpha, \beta)$ corresponding to a specific entrance channel α and a specific exit channel β . Therefore, one specifies the quantum states of all reaction pairs before and after the reaction. Thus:

$\sigma(\alpha, \beta) = \sigma_c(\alpha) G_c(\beta)$ where $\sigma_c(\alpha)$ is the cross-section for the formation of C through channel α , and $G_c(\beta)$ is the probability of C decaying through channel β .⁷

The probability $G_c(\beta)$ that the compound nucleus C will disintegrate through a specific channel β can be written in the form

$$G_c(\beta) = \frac{k \beta^2}{\sum_{\gamma} k \gamma^2} \frac{\sigma_c(\beta)}{\sigma_c(\gamma)}$$

where $k = \chi^{-1}$ and the sum is extended over all channels γ into which the compound nucleus can decay. Thus the

⁶Blatt and Weiskopf: 342.

⁷Blatt and Weiskopf: 342.

cross-section for the reaction $\alpha \rightarrow \beta$ can be calculated if the cross-sections $\sigma_c(\gamma)$ for the formation of the compound system by all possible channels are known and if the Bohr assumption is valid. The probability $G_c(\beta)$ is energy dependent, and its value for a compound nucleus excited to an energy E_c can be obtained from the above expression provided the cross-sections $\sigma_c(\gamma)$ and the wave numbers k_γ are known for those channel energies ϵ_γ which produce the compound nucleus with an energy E_c .⁸

c. Decay of the Compound Nucleus

The kinetic energy in channel β is given by

$\epsilon_\beta = \epsilon_\alpha + Q_{\alpha\beta}$ where $Q_{\alpha\beta}$ is the "Q" value from channel α to channel β . Because most of this energy is the kinetic energy of the emitted particle b and only a small portion is the recoil energy of the residual nucleus Y , one can consider ϵ_β to be the kinetic energy of b , and:

$$\epsilon_\beta = \epsilon_{bY} - E_\beta^*$$

where ϵ_{bY} is given by $\epsilon_{bY} = \epsilon_{bY} = \epsilon + Q_{ab}$ and $\epsilon = \epsilon_\alpha$. ϵ is the kinetic energy in the entrance channel (both a and X are in their ground states). This represents the maximum value of ϵ_β which can occur if the residual

⁸Blatt and Weiskopf: 344-345.

nucleus Y remains in the ground state. E_{β}^* is the excitation energy of Y after the reaction and is the difference between the energy of Y after the reaction and the energy of the ground state of Y. So long as E_{β}^* is less than ϵ_{bY} , to every level E_{β}^* of the residual nucleus there corresponds an energy ϵ_{β} of the emerging particle.⁹

Should the energy be high enough, many levels of the residual nucleus can be excited. The energy distribution of the particle b becomes continuous when the energies E_{β}^* are closer together than the definition of the energy in the incident beam or closer than the energy resolution of the experimental arrangement for detection of the reaction products. The shape of the distribution function $G_b(\epsilon) d\epsilon$ for the particles b emitted between ϵ and $\epsilon + d\epsilon$ is given by:

$$G_b(\epsilon) d\epsilon = \sum_{E < E_{\beta} < E + dE} G_c(\beta)$$

where the sum is extended over the channels β whose energy lies within the energy interval $d\epsilon$. The number of terms in this sum is given by the number of levels of the residual nucleus Y with an excitation energy E_{β}^* between E and $E - d\epsilon$, where $E = \epsilon_{bY} - \epsilon$. One can

⁹Blatt and Weiskopf: 365-366.

call this number $w_Y(E)d\xi$ and $w_Y(E)$ the "level density".

In order to determine the relative intensity distribution of the emerging particles $I_b(\xi)d\xi$, one need consider only the numerator in the expression for the probability

$G_c(\beta)$:

$$I_b(\xi)d\xi = \text{constant } \xi \epsilon_c(\beta) w_Y(\xi_{bY} - \xi)d\xi$$

$\epsilon_c(\beta) = \epsilon_c(\xi)$ is a function of the channel energy $\xi = \xi_\beta$.

The factor replaces the factor k_β^2 .¹⁰

Experimentally, it is impossible to determine the channels by which a particle b left the nucleus or even to determine its energy. One is therefore interested primarily in the cross-section of an (a,b) reaction regardless of the specific channels. Should the energy of the incident particle be high enough to leave the residual nucleus in many different possible excited states, one can express the probability in terms of the level density W. Letting $F_b = \sum_\beta k_\beta^2 \epsilon_c(\beta)$, where the sum extends over all open channels β leading to the emission of b, this sum can be expressed in terms of an integral:

$$F_b = F_b(\xi_{bY}) = \frac{2M}{\hbar^2} \int_0^{\xi_{bY}} \epsilon \beta \epsilon_c(\xi_\beta) w_Y(\xi_{bY} - \xi_\beta) d\xi_\beta$$

$\epsilon_c(\xi_\beta)$ is the cross-section for the formation of the

¹⁰ Blatt and Weiskopf: 367.

compound nucleus by a collision with the energy ξ_β between b and the excited nucleus Y^* , the latter having an excitation energy of $\xi_{\beta Y} - \xi_\beta$. These cross-sections depend only upon the channel energy ξ_β . F_b is a function of the maximum energy $\xi_{b Y}$ which the particle b can acquire and leave the residual nucleus Y in its ground state. If the level densities $w_Y(\xi)$ are known, the function F_b can be calculated for all particle types. Thus the cross-section for an (a,b) reaction can be expressed:

$$\sigma(a,b) = \frac{\xi_c(a) F_b}{\sum_c F_c}$$

where the sum extends over all particle types c that may be emitted in this reaction.¹¹

The average energy E of a system is a monotonically increasing function of the nuclear temperature θ . The function $E(\theta)$ has a vanishing derivative $\left(\frac{dE}{d\theta}\right)_{\theta=0}=0$ for $\theta=0$. If one expands $E(\theta)$ near $\theta=0$ in powers of θ , it must begin at least with a quadratic term. This leads to the following assumptions:

(1) $E = E(\theta)$ permits a power series expansion around $\theta = 0$.

(2) the leading term of this expansion is proportional to θ^2 .

¹¹Blatt and Weiskopf: 369-370.

(3) the higher order terms of θ can be neglected.

One can now write, with $a = \text{constant}$, $E = a\theta^2$, from which:

$$\rho = \int \frac{dE}{\theta(E)} = 2(aE)^{\frac{1}{2}} + \text{constant, and for}$$

the level density:

$$w(E) = C \exp(2\sqrt{aE})$$

This is a statistical formula which applies to a region containing many energy levels. It is a very rough approximate orientation of the general features of the densities of nuclear levels. These level densities can be measured experimentally by comparing the energy distribution of the emerging particles in a nuclear reaction with the results obtained from the theoretical formula.¹²

d. Secondary Nuclear Reactions

In most nuclear reactions of the type $a + X = Y + b$, the residual nucleus Y is left in an excited state. The excess energy is emitted by Y many times in the form of one or more gamma-rays until such time that the nucleus reaches the ground state. In some instances the excitation of Y may be high enough for Y to emit another particle, perhaps a neutron or a proton. One can say

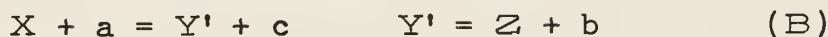
¹²Blatt and Weiskopf: 372.

then that the first nuclear reaction is followed by a "secondary nuclear reaction":

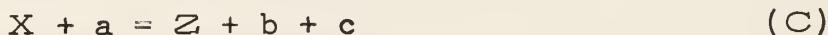


in which a particle c is emitted. The observed reaction is a bombardment of X by a , followed by an emission of b and c leaving a residual nucleus Z . One can represent this process as an $X(a;bc)Z$ reaction. The most common reactions of this type are the $(a;2n)$ reactions.¹³

Observed $X(a;bc)Z$ reactions may be caused by two different mechanisms:



If the time between the emission of the first and the second particle should become very short (of the order of the transit time of nucleons over nuclear dimensions), one can no longer refer to a defined excited state Y or Y' between the two emissions. The reaction should then be described:



This third process constitutes a simultaneous breakup of the compound system C into a nucleus Z and two particles b and c . Process (C) does not correspond to any of the

¹³Blatt and Weiskopf: 374.

"channels" which have been used so far. Blatt and Weiskopf indicate that an assumption should be made that the reaction $X(a;bc)Z$ does not go to process (C). There is some doubt as to whether or not this reaction should be included; however, in the computer calculations which follow, this assumption is not made and the process (C) is included.¹⁴

The cross-section for process (A) is denoted by $\epsilon(a;bc)$ and the cross-section for process (B) by $\epsilon(a;c,b)$. The observed cross-section of the $(a;bc)$ reaction will be called $\bar{\epsilon}(a;bc)$. It is the sum of the cross-sections for processes (A) and (B):

$$\bar{\epsilon}(a;bc) = \epsilon(a;b,c) + \epsilon(a;c,b)$$

Usually one of these two contributions is much more important than the other, but there may be cases in which $\epsilon(a;b,c)$ and $\epsilon(a;c,b)$ are of the same order of magnitude. The distinction between processes (A) and (B) should not be made if b and c are the same kind of particle (e.g., $(a;nn)$ reactions).¹⁵

Considering the definition

$$\epsilon(a,b) = \epsilon_C(a)G_C(b)$$

¹⁴Blatt and Weiskopf: 374-375.

¹⁵Blatt and Weiskopf: 375.

the cross-section $\sigma(a,b)$ also includes those reactions in which the residual nucleus undergoes a secondary reaction at a later time. One can therefore distinguish the cross-section $\sigma^*(a,b)$ which constitutes only those reactions where the residual nucleus does not emit additional particles. $\sigma^*(a,b)$ is what is understood conventionally by the cross-section of an $X(a,b)Y$ reaction. The following relation holds:

$$\sigma(a,b) = \sigma^*(a,b) + \sum_c \sigma(a;b,c)$$

One can express the cross-section for the $(a;b,c)$ reaction (process (A)) as follows:

$$\sigma(a;b,c) = \sigma_C(\alpha) \sum_{\beta} G_c(\beta) G_{Y\beta}(c)$$

α is the entrance channel, and the prime on the sum indicates that it is extended only over those channels β in which the residual nucleus Y is excited enough to emit a particle c . $G_c(\beta)$ is the relative probability (a pure number) that the compound nucleus C will decay into channel β , while $G_{Y\beta}(c)$ is the relative probability that the nucleus Y in state β' (corresponding to channel β) will emit a particle c . ¹⁶

The simplifying assumption can be made that the residual nucleus Y always emits a particle if its excitation

¹⁶Blatt and Weiskopf: 376.

energy is high enough. Let $S_{\min}(Y)$ be the smallest separation energy of a particle from Y. The assumption is then expressed by:

$$\sum_c G_{Y\beta'}(c) \cong 1 \text{ provided that } E_{bY} - E_\beta \geq S_{\min}(Y).$$

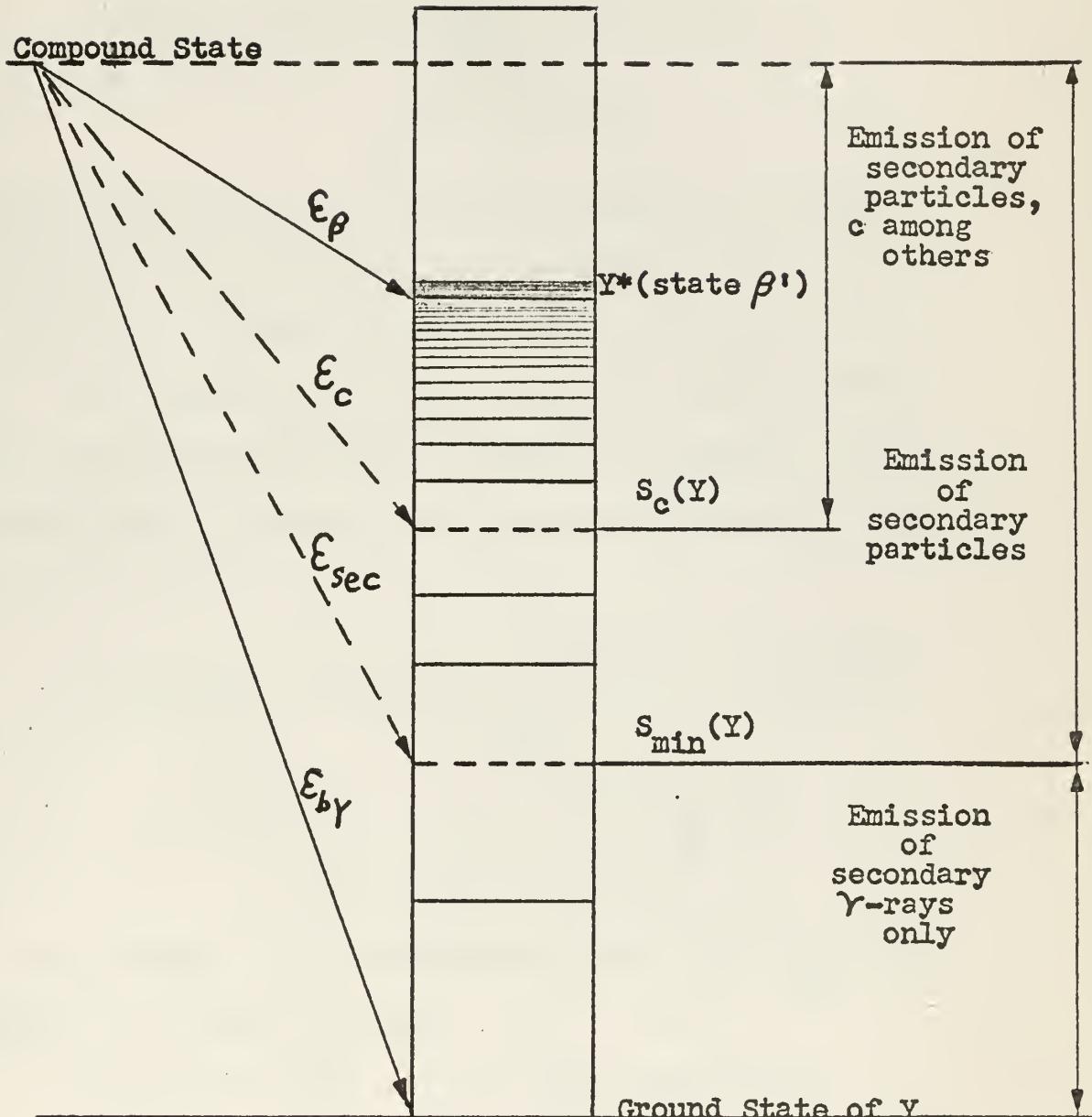
This is the equivalent of neglecting the emission of gamma-rays in competition with the emission of particles, an assumption which is approximately correct in most cases except in the vicinity of the threshold. One can thus write:

$$\sigma^*(a,b) \cong \sum_{\beta}'' \sigma(a,\beta)$$

where the double prime on the sum indicates that it is extended only over those channels in which the residual nucleus Y is excited below $S_{\min}(Y)$. Let E_{\sec} be the maximum energy available for emission of secondary particles from the first residual nucleus Y for a given excitation energy in the original compound nucleus. This energy is also the energy of the emerging particles b in the primary $X(a,b)Y$ reaction at which Y can just begin emitting particles (See Figure 1). E_{\sec} is given by:

$$E_{\sec} = E_{bY} - S_{\min}(Y)$$

One then obtains the following expressions for the "observed" cross-section $\sigma^*(a,b)$ of the $X(a,b)Y$ reaction:



Energy Relationships in the Reaction



FIGURE 1¹⁸

$$\sigma^*(a,b) = \sigma(a,b) \quad (\text{for } \epsilon_{\text{sec}} < 0)$$

$$\sigma^*(a,b) \approx \sigma(a,b) \frac{\int_{\epsilon_{\text{sec}}}^{\epsilon_{bY}} I_b(\epsilon) d\epsilon}{\int_0^{\epsilon_{bY}} I_b(\epsilon) d\epsilon} \quad (\text{for } \epsilon > 0)$$

Where $I_b(\epsilon) d\epsilon$ is the relative probability of emitting particles b with energies ϵ between ϵ and $\epsilon + d\epsilon$.¹⁷
 $(I_b(\epsilon) d\epsilon = \text{constant} \epsilon \epsilon_C(\beta) W_Y(\epsilon_{bY} - \epsilon) d\epsilon)$.¹⁸

The cross-section for a secondary nuclear reaction with emission of particles c following a primary (a,n) reaction can be written with the same approximation as:

$$\sigma(a;n,c) \cong \sigma(a,n) \frac{\int_0^{\epsilon_c} \exp(-\frac{\epsilon}{\theta}) G_{Y\beta}(c) d\epsilon}{\int_0^{\epsilon_{bY}} \exp(-\frac{\epsilon}{\theta}) d\epsilon}$$

$$\epsilon_c \equiv \epsilon_{bY} - S_c(Y)$$

In this formula ϵ_c is the excess energy over the threshold of the (a;n,n) reaction. $S_c(Y) = S_n(Y)$.²⁰

Should the energy of the incident particle be sufficiently large, the residual nucleus after emission of the second particle (nucleus Z) may be excited enough to

¹⁷Blatt and Weiskopf: 376.

¹⁸Blatt and Weiskopf: 377.

¹⁹Blatt and Weiskopf: 377.

²⁰Blatt and Weiskopf: 378-379.

emit a third particle. One would then observe tertiary reactions or reactions of even higher order.²¹

3. The Statistical Model

The model portrayed herein is a statistical representation of the nucleus and has been adapted for calculations in a digital computer. This model, with due regard to the appropriate assumptions, is general and can be used to make statistical predictions regarding the cross-section for the emission of particles from any initial nucleus. In its current status, the computer program is somewhat limited in that the following nuclear properties have been neglected:

- a. The coulomb barrier
- b. Spin
- c. Parity
- d. Angular momentum
- e. Tertiary and higher order reactions

The density function $P(E)$ is the same for the different product nuclei.

Tertiary reactions were considered, but have not been included in the main program. There is included in Appendix II a computer subroutine which can be used to

²¹Blatt and Weiskopf: 378

compute tertiary reactions.

The coulomb force can affect the neutron processes indirectly. Should a proton process be made smaller, then the neutron process will be made larger. In the calculations studied herein, the coulomb effects are neglected.

In setting up the model and fitting it to the computer program, Cu⁶⁵ was selected as the experimental nucleus with which to compare the theoretical results. Cu⁶⁵ was selected for the following reasons:

- a. Cu⁶⁵ is a light nucleus and is less complicated than others.
- b. Cu⁶⁵ has a low coulomb barrier.
- c. Experimental data is available for making desired comparisons with theoretical calculations.

The experimental data was taken from graphs of the published results of experiments with photo neutron cross-sections conducted at the Ernest O. Lawrence Radiation Laboratory, Livermore, California in August 1963. (See Figures 2,3 and 4) In the Livermore experiments the single neutron cross-sections actually consist of the sum of the reactions:

$$\sigma_n + \sigma_{np} + \sigma_{n2P} + \dots$$

and the double neutron reactions:

$$\sigma_{2n} + \sigma_{2np} + \sigma_{2n2p} + \dots$$

The general technique here is to compute the photo neutron cross-section for the $X(\gamma, 2n)Y$ process and compare it with the experimental data for the same process. This is an application of the relationship:

$$\sigma(a;n,c) \cong \sigma(a,n) \frac{\int_0^{\xi_c} \exp(-\frac{\xi}{\theta}) G_{Y\beta}(c) d\xi}{\int_0^{\xi_{bY}} \exp(-\frac{\xi}{\theta}) d\xi}$$

$$\xi_c = \xi_{bY} - S_c(Y)$$

$\sigma(a,n)$ is taken from the experimental data for Cu^{65} and is used throughout in all computer calculations. For these processes, $\sigma(a,n)$ is the $\sigma(\gamma,n)$, and $\sigma(a;n,c)$ is $\sigma(\gamma;2n)$.

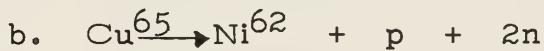
The range of gamma-ray energy for which calculations are made extends from 0 to 30 MEV. Within this energy range, six reactions are possible. These reactions are portrayed in Figure 5.

In order to make computer calculations less complicated, channel widths are set at 0.3 MEV, and all integrals are calculated in accordance with this incremental change in energy. Also, the threshold energies for each reaction have been adjusted so as to be divisible by 0.3 for ease in carrying out the necessary numerical integration.

The probability of decay to any particular state is

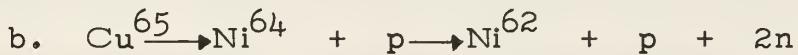
statistical, and is computed for both single-step and two-step processes. In setting up the model, the return of the compound nucleus to its ground state by the emission of a gamma-ray is neglected.

From Figure 5, it is noted that there are two direct processes by which two neutrons can be emitted:

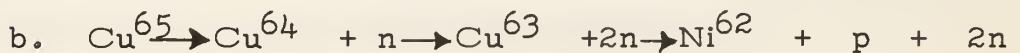
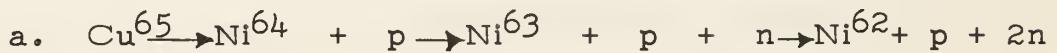


For simplicity, the $\text{Cu}^{65} (\gamma; p, 2n) \text{Ni}^{62}$ process will be omitted. These omitted processes contribute to the overall cross-section only at energies above their thresholds. For this process, the threshold is taken to be 23.7 MEV.

Four secondary processes result in the emission of two neutrons:



Two tertiary processes result in the emission of two neutrons:



In each case, the tertiary processes and the Cu^{65}
 $(\gamma; p, 2n) \text{Ni}^{62}$ processes are omitted since they contribute
to the overall cross-section only at energies above the
threshold of 23.7 MEV.

Single neutron emitting processes are as follows:

Three direct

- a. $\text{Cu}^{65} \rightarrow \text{Cu}^{64} + n$
- b. $\text{Cu}^{65} \rightarrow \text{Ni}^{63} + p + n$
- c. $\text{Cu}^{65} \rightarrow \text{Co}^{62} + 2p + n$

The $(\gamma; 2p, n)$ process is omitted.

Three secondary

- a. $\text{Cu}^{65} \rightarrow \text{Ni}^{64} + p \rightarrow \text{Ni}^{63} + p + n$
- b. $\text{Cu}^{65} \rightarrow \text{Ni}^{64} + p \rightarrow \text{Co}^{62} + 2p + n$
- c. $\text{Cu}^{65} \rightarrow \text{Cu}^{64} + n \rightarrow \text{Ni}^{63} + p + n$

The $(\gamma; 2p, n)$ process is omitted. All tertiary single
neutron processes are omitted.

The decay of the compound nucleus for each process
may be expressed as an integral of the density function
integrated over all available energy channels:

$$\int_0^{E-E_{\text{Ethl}}} P(E') dE'$$

This integral represents the decay of Cu^{65} to $\text{Ni}^{64} + p$.

Other reactions are represented accordingly.

Probabilities for direct processes are calculated as follows:

a. Two-neutron processes

$$(1) \text{ Cu}^{65} \rightarrow \text{Cu}^{63} + 2n$$

$$E - E_{\text{Eth4}}$$

$$P_1(E) = \frac{\int_0^E R(E - E_{\text{Eth5}}) P(E') dE'}{\int_0^{E_{\text{Eth1}}} P(E') dE' + \int_0^{E_{\text{Eth2}}} P(E') dE' + \int_0^{E_{\text{Eth3}}} P(E') dE' + \int_0^{E_{\text{Eth4}}} P(E') dE' + \int_0^{E_{\text{Eth5}}} P(E') dE' + \int_0^{E_{\text{Eth6}}} P(E') dE'}$$

Where $P_1(E)$ is the probability of decay of the compound nucleus C^* to Cu^{63} with the emission of two neutrons.

The function $R(E - E_{\text{Eth5}})$ will be explained in subsequent discussion.

$$(2) \text{ Cu}^{65} \rightarrow \text{Ni}^{62} + p + 2n$$

$$E - E_{\text{Eth5}}$$

$$P_2(E) = \frac{\int_0^E P(E') dE'}{\int_0^{E_{\text{Eth1}}} P(E') dE' + \int_0^{E_{\text{Eth2}}} P(E') dE' + \int_0^{E_{\text{Eth3}}} P(E') dE' + \int_0^{E_{\text{Eth4}}} P(E') dE' + \int_0^{E_{\text{Eth5}}} P(E') dE' + \int_0^{E_{\text{Eth6}}} P(E') dE'}$$

b. Single neutron processes

$$(1) \text{ Cu}^{65} \rightarrow \text{Cu}^{64} + n$$

$$E - E_{\text{Eth2}}$$

$$R(E - E_{\text{Eth5}}) E(E') dE'$$

$$P_3(E) = \frac{\int_0^E R(E - E_{\text{Eth5}}) E(E') dE'}{\int_0^{E_{\text{Eth1}}} P(E') dE' + \int_0^{E_{\text{Eth2}}} P(E') dE' + \int_0^{E_{\text{Eth3}}} P(E') dE' + \int_0^{E_{\text{Eth4}}} P(E') dE' + \int_0^{E_{\text{Eth5}}} P(E') dE' + \int_0^{E_{\text{Eth6}}} P(E') dE'}$$

$$(2) \text{ Cu}^{65} \rightarrow \text{Ni}^{63} + p + n$$

E-Eth3

$$P_4(E) = \frac{\int_0^E R(E - E_{\text{th}}) F(E') dE'}{\int_0^{E_{\text{th}1}} P(E') dE' + \int_0^{E_{\text{th}2}} P(E') dE' + \int_0^{E_{\text{th}3}} P(E') dE' + \int_0^{E_{\text{th}4}} P(E') dE' + \int_0^{E_{\text{th}5}} P(E') dE' + \int_0^{E_{\text{th}6}} P(E') dE'}$$



The probabilities for secondary processes are calculated with a slightly different procedure. These two-step processes consist of a decay of the compound nucleus to a second excited nucleus. This second excited nucleus then decays by emitting a particle. Thus the probability for the complete process will be formed by the product of the probabilities of two independent reactions. For the general scheme of decay, consider a hypothetical example. (See Figure 6)

The available energy of the initial reaction ranges between 0 and 30 MEV and is represented by E . Decay of the compound nucleus to state A can occur if E is greater than the threshold energy $E_{\text{th}A}$. Likewise, if E is greater than $E_{\text{th}B}$, the compound nucleus may also decay to state B. Should this condition prevail, then both states A and B will compete for the reaction.

Probabilities for the two-step process must then make provisions for the consideration of this competition. Now, once the initial reaction has taken place and a second compound nucleus has been formed, this second compound nucleus will tend to decay if the energy is high enough. Thus, if $E - EtA$ is greater than $EtB - EtA$, this second compound nucleus will tend to decay. The decay of this second compound nucleus by the emission of a particle will cause a decrease in the probability of this second state existing when the energy exceeds the value of $E - EtB$. This "leakage" must be compensated for when the probabilities are calculated.

In calculating the probabilities for two-step reactions, the reaction which occurs second physically must be computed first mathematically. To compensate for the "leakage" factor, the function $R(E - Eth)$ is defined as follows:

$$R(E - Eth) = 0 \quad \text{for } E - Eth > 0$$

$$R(E - Eth) = 1 \quad \text{for } E - Eth < 0$$

The particular value of Eth will depend upon the reaction. The function $R(E - Eth)$ becomes a part of the density function for the second reaction and is integrated along with the density function.

The "leakage" must also be considered for the single neutron and single proton processes for energy values above the two-neutron threshold since the two-neutron process will "rob" the single neutron process. This "leakage" is compensated for in the appropriate equations by the same function $R(E - E_{th})$.

Consider the two-step reaction:



Where X represents the initial compound nucleus, Y represents state A, and Z represents state B. Once state B has been formed, this state will exist until the energy E_2 exceeds the value $E_{tC} - E_{tB}$, at which energy there will tend to be a "leakage" of particles from state B to state C (provided of course that state C is one which is physically possible from state B). This "leakage" is compensated for by the function $R(E - E_{th})$. Thus the integral for the second of the two processes would be:

$$P_{AC}(E) = \int_0^{E - E_{tA} - (E_{tB} - E_{tA})} R(E' + E_{tA} - E_{tB}) P(E') dE'$$

But, $E - E_{tA} - (E_{tB} - E_{tA}) = E - E_{tB}$, and $E_{tI} + E_{tA} = E$. Therefore:

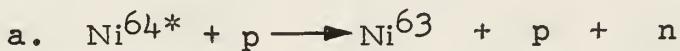
$$P_{AC}(E) = \int_0^{E - E_{tB}} R(E - E_{tB}) R(E') dE'$$

The integral for the complete two-step process, combined with the appropriate denominator, would appear thusly:

$$P_{OAC}(E) = \frac{\int_0^E F(E')}{D1} \left[\frac{\int_0^E P_{AC}(E') dE' dE'}{D2} \right]$$

D1 represents the denominator for the decay of the compound nucleus to state A, and D2 represents the denominator for the decay of state A to state B.

Using the above procedure, the probabilities for the two-step processes can be written. Consider first the secondary reactions:



$$R(E - Eth5) P(E') dE'$$

$$P_{2pn}(E') = \frac{\int_0^E P(E') dE'}{E - Eth3} \left[\frac{\int_0^E P(E') dE'}{E - Eth5} + \frac{\int_0^E F(E') dE'}{E - Eth6} \right]$$



$$R(E - Eth5) P(E2') dE2'$$

$$P_{2np}(E2') = \frac{\int_0^E P(E2') dE2'}{E - Eth3} \left[\frac{\int_0^E F(E2') dE2'}{E - Eth5} + \frac{\int_0^E P(E2') dE2'}{E - Eth6} \right]$$

Now, the probabilities for the two-step single-neutron processes can be written:



$$\begin{aligned} P_{2pn}(E) = & \frac{\int_0^E P(E') dE' \int_0^{E'} P_{2pn}(E') dE'}{\int_0^{E-Eth1} P(E') dE' + \int_0^{E-Eth2} P(E') dE' + \int_0^{E-Eth3} P(E') dE' + \int_0^{E-Eth4} P(E') dE' + \int_0^{E-Eth5} P(E') dE' + \int_0^{E-Eth6} P(E') dE'} \\ & \text{E - Eth1} \quad \text{E - Eth3} \\ & P(E') \quad P_{2pn}(E') dE' dE' \end{aligned}$$



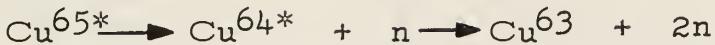
$$\begin{aligned} P_{2np}(E) = & \frac{\int_0^E P(E') dE' \int_0^{E'} P_{2np}(E') dE'}{\int_0^{E-Eth1} P(E') dE' + \int_0^{E-Eth2} P(E') dE' + \int_0^{E-Eth3} P(E') dE' + \int_0^{E-Eth4} P(E') dE' + \int_0^{E-Eth5} P(E') dE' + \int_0^{E-Eth6} P(E') dE'} \\ & \text{E - Eth2} \quad \text{E - Eth3} \\ & P(E') \quad P_{2np}(E') dE' dE' \end{aligned}$$

The probability for the two-step two-neutron process can be obtained similarly. Again consider first the secondary reaction:



$$P_{2nn}(E2) = \frac{R(E - Eth5) P(E2') dE2'}{\int_0^{E-Eth3} P(E2') dE2' + \int_0^{E-Eth4} P(E2') dE2' + \int_0^{E-Eth5} P(E2') dE2' + \int_0^{E-Eth6} P(E2') dE2'}$$

The probability for the complete two-step process becomes:



$$P_{2nn}(E) = \frac{\int_0^E P(E') dE' + \int_0^E P(E') dE'}{P(E-Eth1) + P(E-Eth2) + P(E-Eth3) + P(E-Eth4) + P(E-Eth5) + P(E-Eth6)}$$

$E - E_{\text{Eth}2}$ $E - E_{\text{Eth}4}$
 $P(E')$ $P_{2nn}(E)$ dE dE'

The total probability for each process is the sum of the probabilities for the individual reactions since these are mutually exclusive events. The total probability for the two-neutron processes can be written:

$$P_{nn}(E) = P_{2nn}(E) + P_1(E) + P_2(E)$$

The total probability for all single-neutron processes is:

$$P_n(E) = P_{2pn}(E) + P_{2np}(E) + P_3(E) + P_4(E)$$

Statistically, the ratio of the cross-sections for the two-neutron processes to the cross-sections for the single-neutron processes should equal the ratio of corresponding probabilities:

$$\frac{\sigma(\gamma, 2n)}{\sigma(\gamma, n)} = \frac{P(\gamma, 2n)}{P(\gamma, n)}$$

and

$$\sigma(\gamma, 2n) = \sigma(\gamma, n) \left[\frac{P(\gamma, 2n)}{P(\gamma, n)} \right]$$

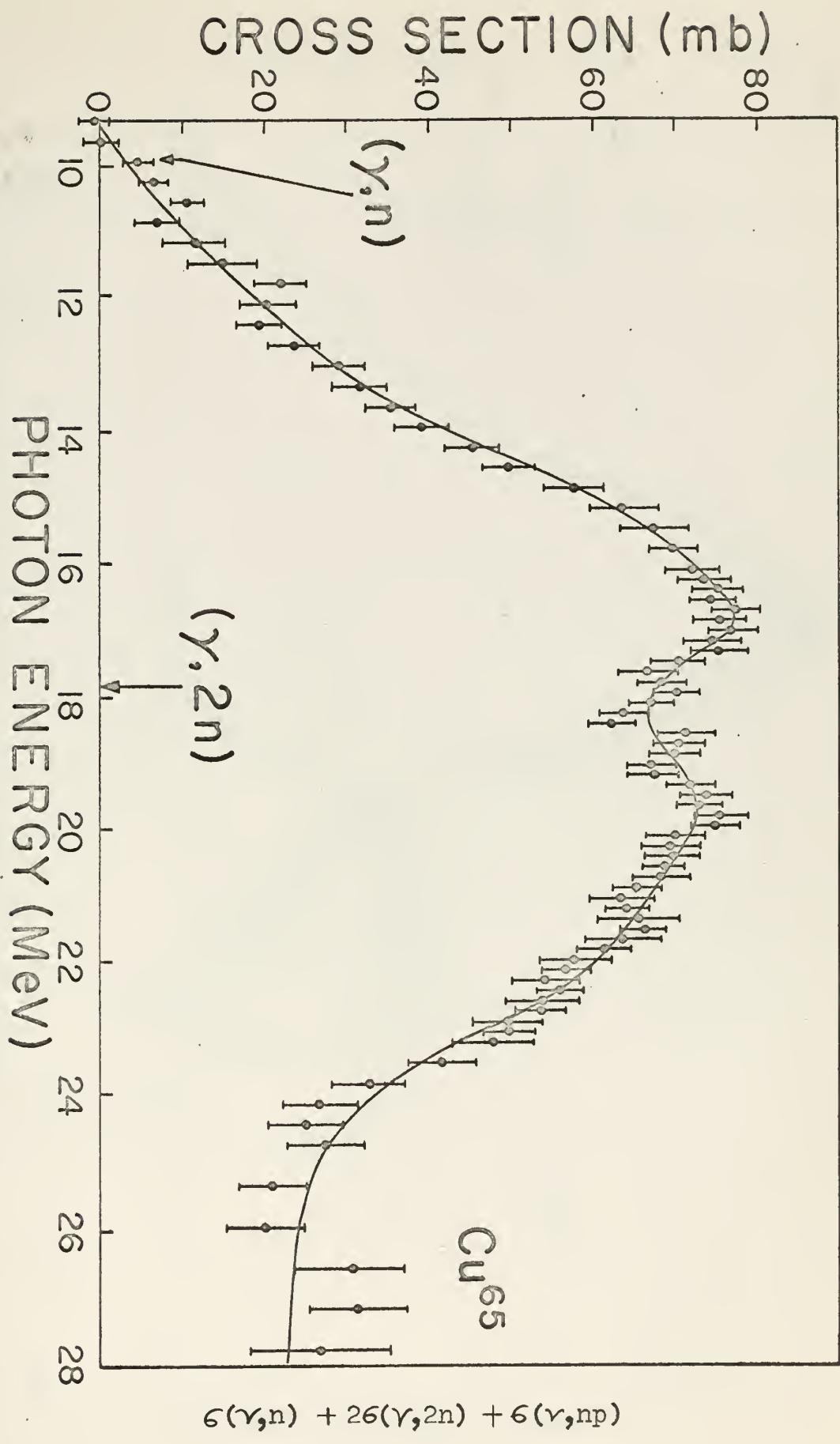


FIGURE 2²²

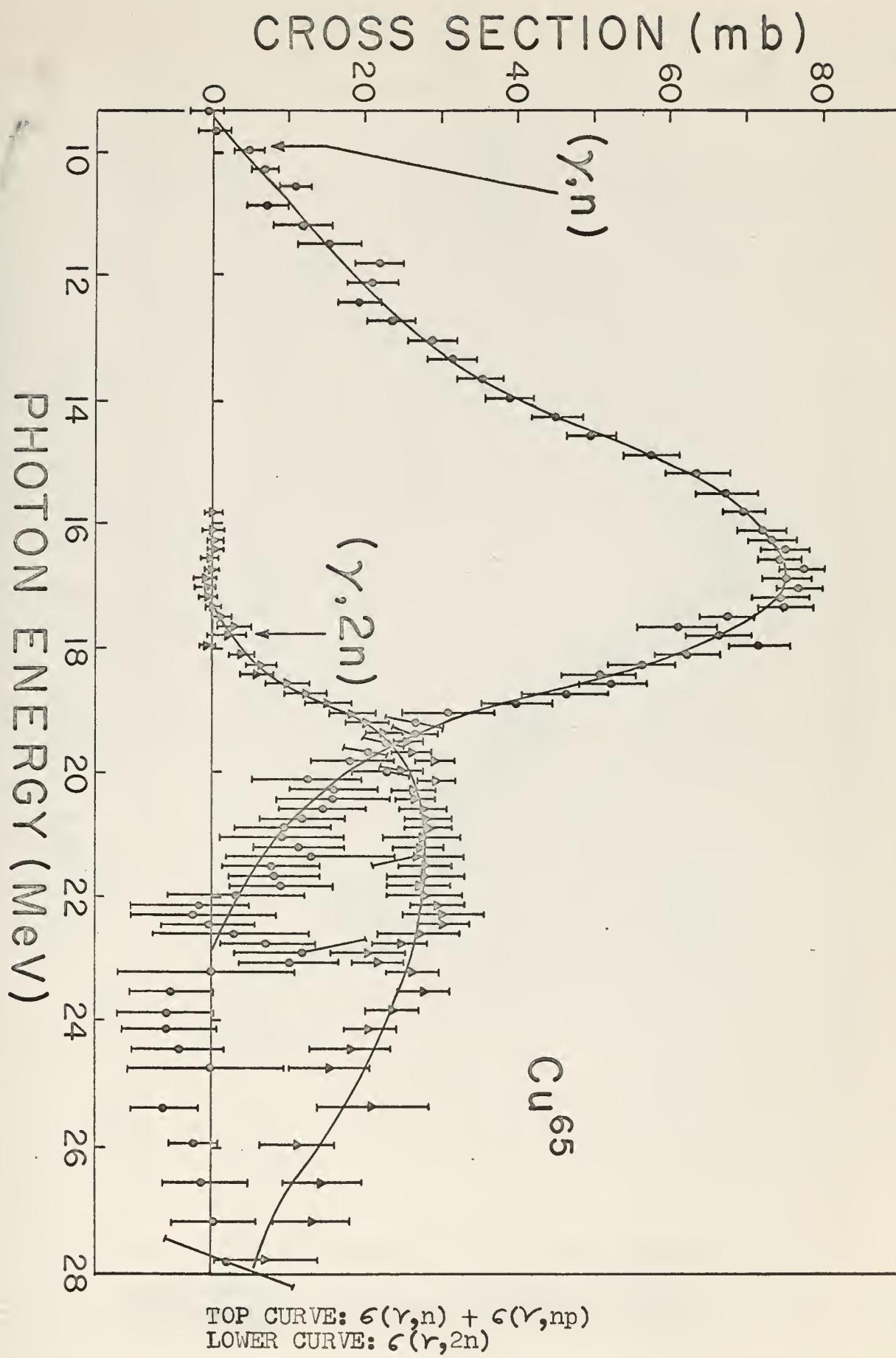


FIGURE 3²³

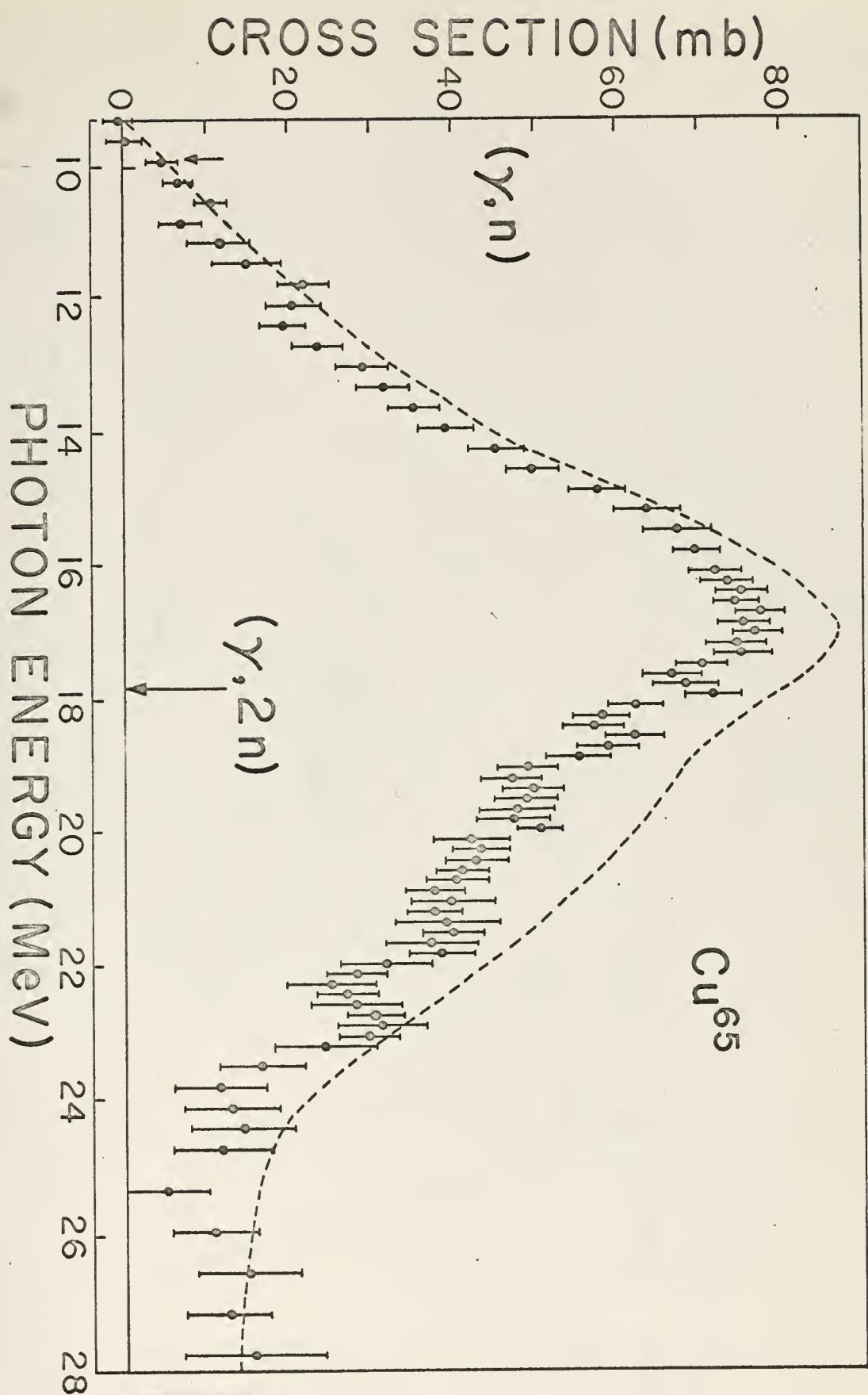
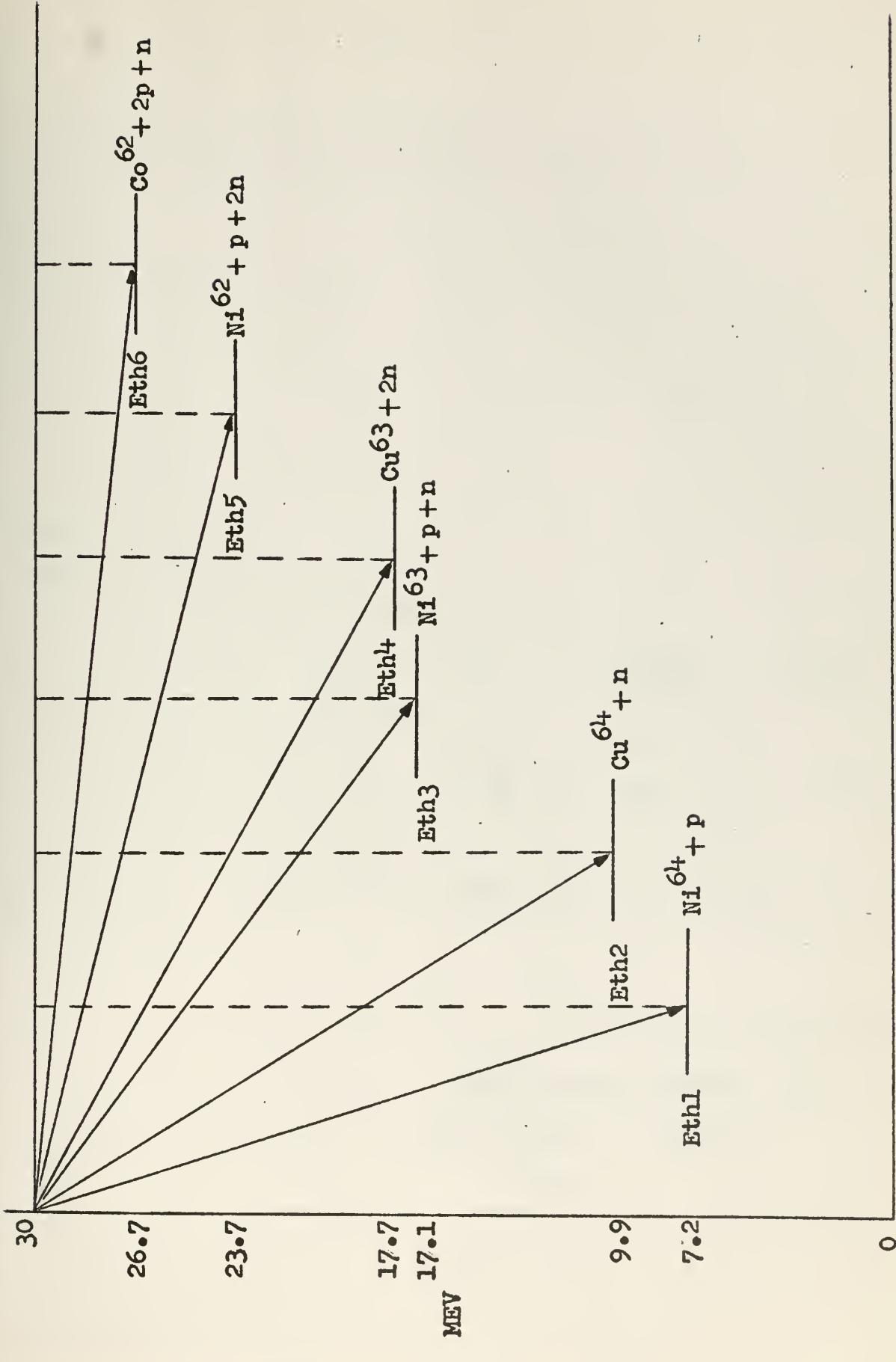
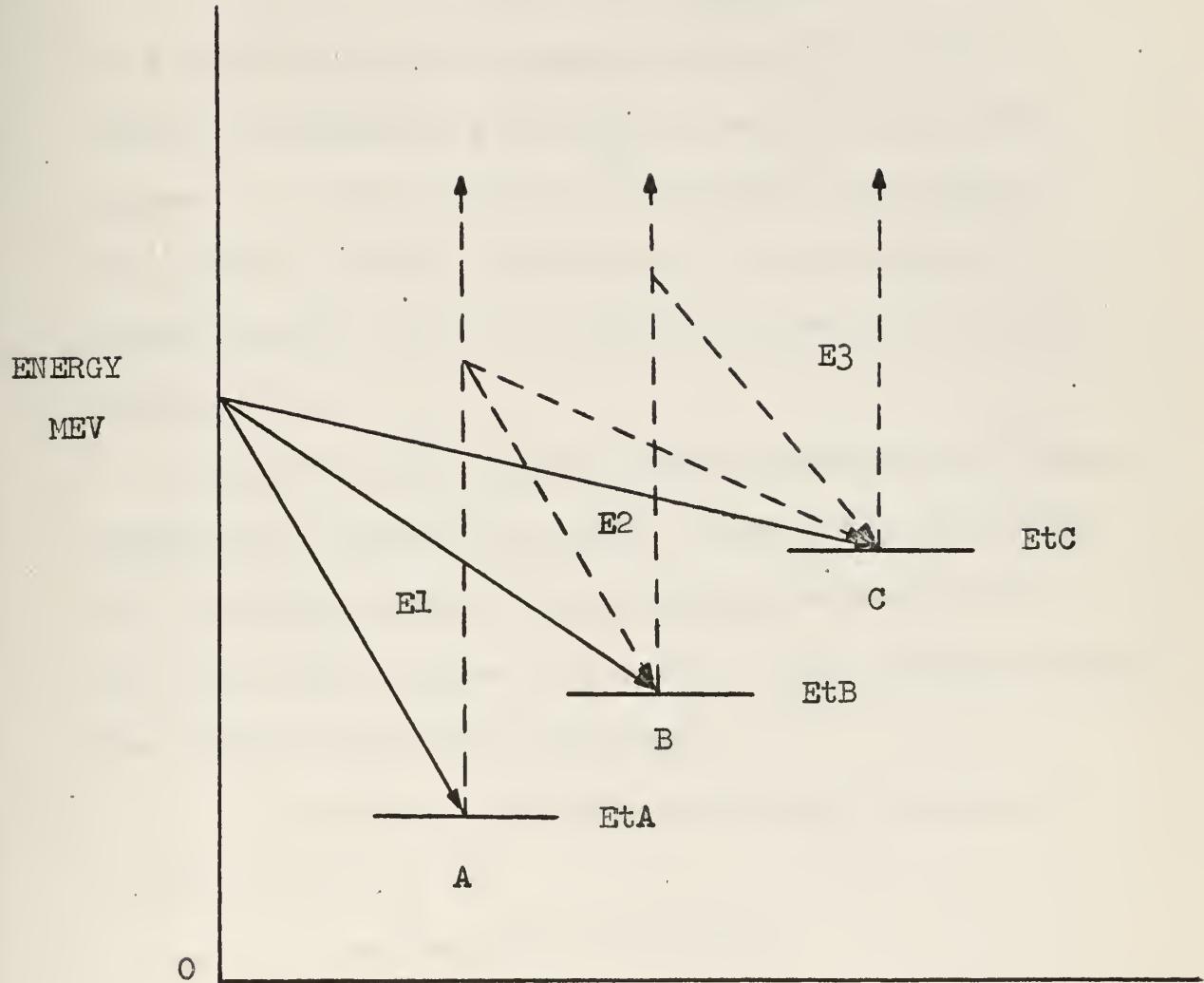


FIGURE 4²⁴



Energy Level Diagram

Figure 5



ENERGY LEVEL DIAGRAM
HYPOTHETICAL EXAMPLE
FIGURE 6

where $P(\gamma, 2n) = P_{nn}(E)$ and $P(\gamma, n) = P_n(E)$. This relationship is the basis on which the computer program is written.

4. Computational Parameters

Blatt and Weiskopf present the function

$$P(E) = C \exp(2\sqrt{AE})$$

as a first guess in the determination of the density function describing the statistical decay of the compound nucleus. In this function, E represents the energy of the incident particle (gamma-ray); C is a constant of proportionality; and A is a constant based on the nuclear temperature 0.

From this first guess, several variations have been devised in an attempt to obtain a density function which will closely approximate the available experimental data for the $(\gamma, 2n)$ process. In each of these density functions the following constants are used:

A = Nuclear temperature constant; the units of A are $(\text{MEV})^{-1}$

C = Constant of proportionality

²²Fultz, S.C., Bramblett, R.L., Caldwell, J.T., and Harvey, R.R., Photo-Neutron Cross-sections for Natural Cu, Cu⁶³, and Cu⁶⁵. Physical Review, v. 133, no. 5B, 9 March 1964: B1149 - B1154.

²³Fultz, Bramblett, Caldwell and Harvey: B1149-B1154.

²⁴Fultz, Bramblett, Caldwell and Harvey: B1149-B1154.

$n = RN$ = Exponent of the (AE) term

Altogether, fourteen density functions have been formulated for this program (See Appendix III); however, only six have been investigated in any detail. These six are:

- a. $P(E) = C(Z - E) \exp(A(E^n - Z^n))$; $Z = E - E_{th}$
- b. $P(E) = C \exp(A(E^n))$
- c. $R(E) = CZ \exp(A(E^n))$
- d. $P(E) = CE \exp(A(E^n))$
- e. $P(E) = CE/\exp(A(E^n))$
- f. $P(E) = C(E)^3 \exp(A(E^n))$

Each of these density functions has been tested with various values of A. In all cases, the parameter $n = RN$ was set equal to 1.0. The values of A ranged between 0.0001 and $10.0 (\text{MEV})^{-1}$.

For each of the values of A, the cross-sections for the theoretical ($\gamma, 2n$) process were plotted versus the energy of the incident gamma-ray. On the same graph in each case were plotted the experimental cross-sections for the corresponding values of gamma-ray energies. The procedure used is to compare the theoretical cross-sections with the experimental values. The objective here is to find a density function $P(E)$ that will give a close

approximation to the experimentally determined cross-sections.

5. Results

a. $P(E) = C(Z - E) \exp(-A(E^n - Z^n))$; $Z=E - E_{\text{th}}$.

See Figures 7 through 14. The idea for this function was obtained from Ericson's article on The Statistical Model and Nuclear Level Densities. At values of A ranging from 0.0001 to 1.0 $(\text{MEV})^{-1}$, it shows a shape that in general conforms to the shape of the experimental curve; however, for larger values of A the curve develops a spike near the lower energy values. Thus, this function is not considered to be appropriate.

b. $F(E) = C \exp(-A(E^n))$. See Figures 15 through 23. This function is a modification of the one suggested by Blatt and Weiskopf as a first guess. This function was tested for values of A ranging from 0.0005 to 10.0 $(\text{MEV})^{-1}$. This function reached its maximum value at an A value of about 0.1 $(\text{MEV})^{-1}$ and held this value for all decreasing values of A. As the value of A increased from 0.1 $(\text{MEV})^{-1}$, the function decreased in value. The values became so small at an A value of about 2.0 $(\text{MEV})^{-1}$ that the curve did not even show up. In all instances, a small spike appeared on the lower energy

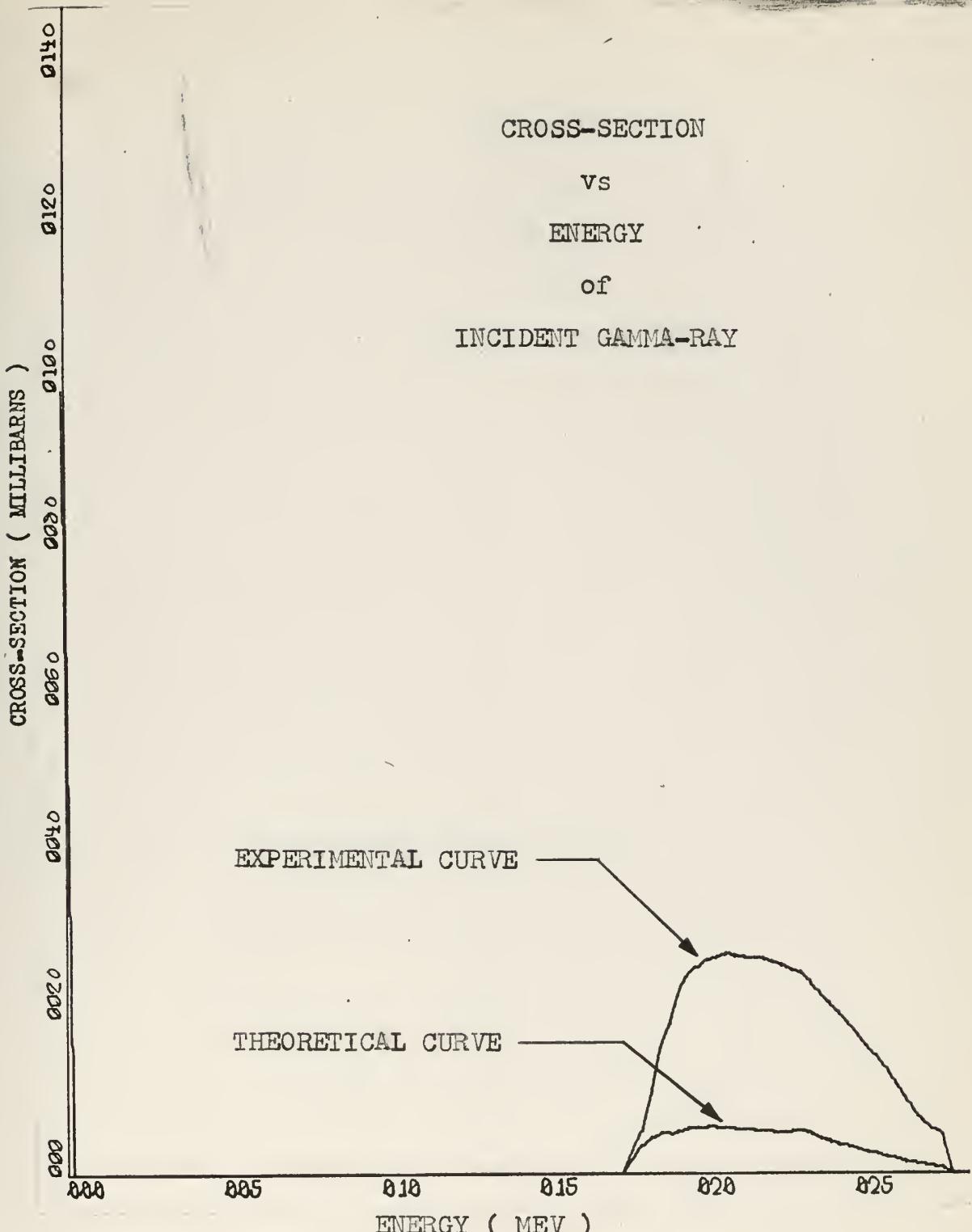
end of the curve; however, it was not as pronounced as the spike obtained with the function in a. above. This function is not considered to be descriptive of the experimental cross-section.

c. $R(E) = CZ \exp(A E^n)$. See Figures 24 through 33. For this function, A ranged in value from 0.0001 to 10.0 (MEV)⁻¹. Throughout the entire range of A values, the shape of the curve was generally in conformity with the experimental curve except that the ordinates were at all times much too small. This function reached its maximum value at an A value of about 0.5 (MEV)⁻¹. For increasing values, the curve became unobservable at an A of about 1.5 (MEV)⁻¹. This function gives a good shape, but the values of the cross-section are too small.

d. $R(E) = CE \exp(A(E^n))$. See Figures 34 through 49. Values of A ranged from 0.0001 to 10.0 (MEV)⁻¹. The shape of the curve was good throughout, but its maximum value never exceeded about one-fourth of the experimentally determined values. It reached its maximum value at an A of about 0.01 (MEV)⁻¹; it became too small to observe at an A of about 1.5 (MEV)⁻¹. This function does not have a high enough maximum value.

e. $P(E) = CE/\text{EXP}(A(E^n))$. See Figures 50 through 56. The values of A ranged from 0.0001 to $10.0 (\text{MEV})^{-1}$ for this function. Throughout this range of values, the curve was visible and had significant magnitude. However, there developed a spike on the low energy end which became quite pronounced as the values of A increased. This is not considered to be an appropriate density function.

f. $H(E) = C(E)^3 \text{ EXP}(A(E^n))$. See Figures 57 through 64. This function was devised in an attempt to increase the magnitude of the ordinates for the cross-section curve. From the figures, it can be seen that instead of increasing the magnitudes of the ordinates, it actually decreased them. For this function, values of A ranged from 0.0001 to $3.0 (\text{MEV})^{-1}$. Throughout most of this interval, the curve held a good shape. The magnitudes became so small that as the values of A increased through $1.0 (\text{MEV})^{-1}$ the curve was no longer visible. Although the energy coefficient $(E)^3$ does not adequately describe the process, it does suggest the possibility that perhaps some other power of (E) may give the curve a better shape.

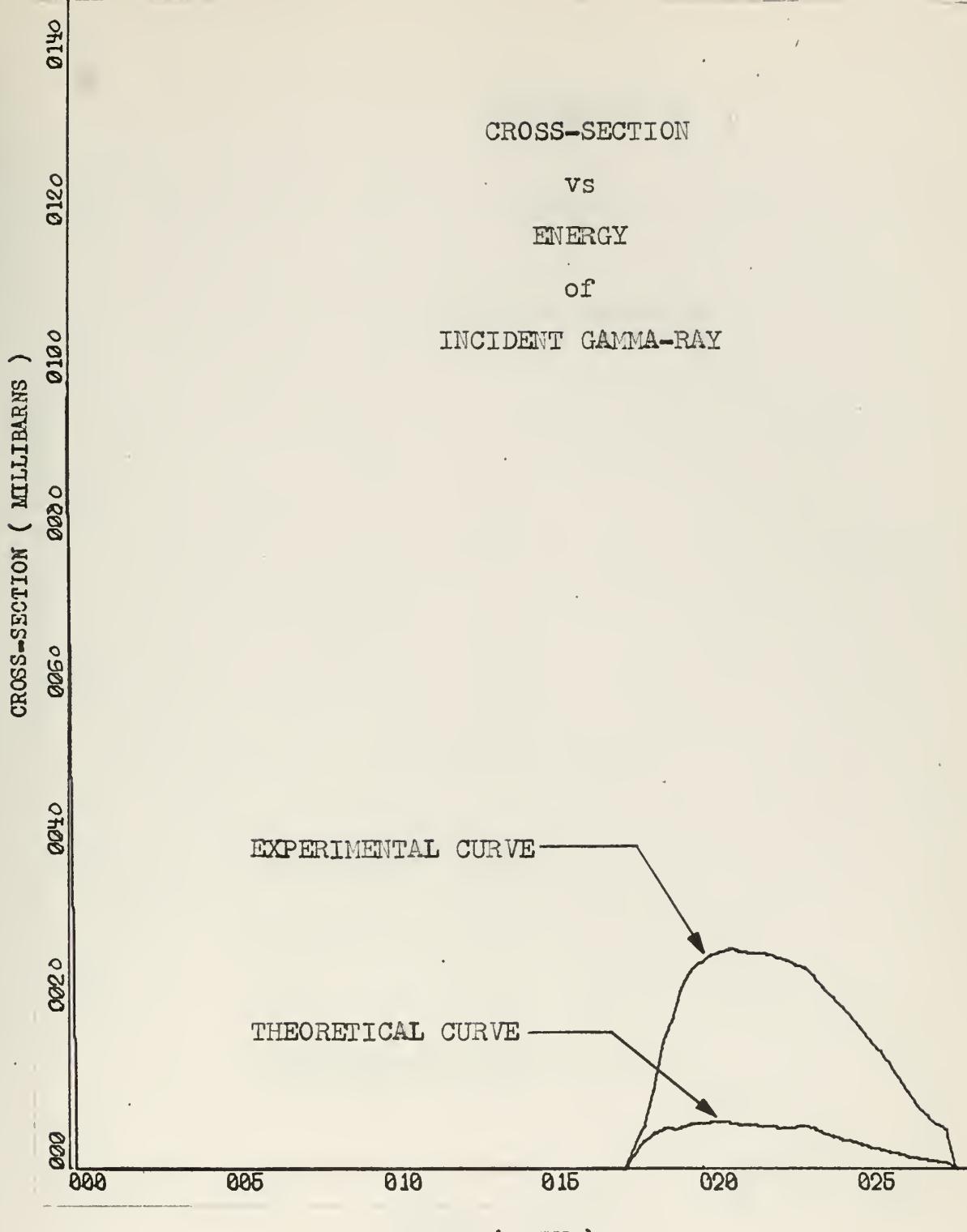


DENSITY FUNCTION $P(E) = C(Z - E) \exp(A(E^n - Z^n))$

$$A = 0.0001 \text{ MEV}^{-1}$$

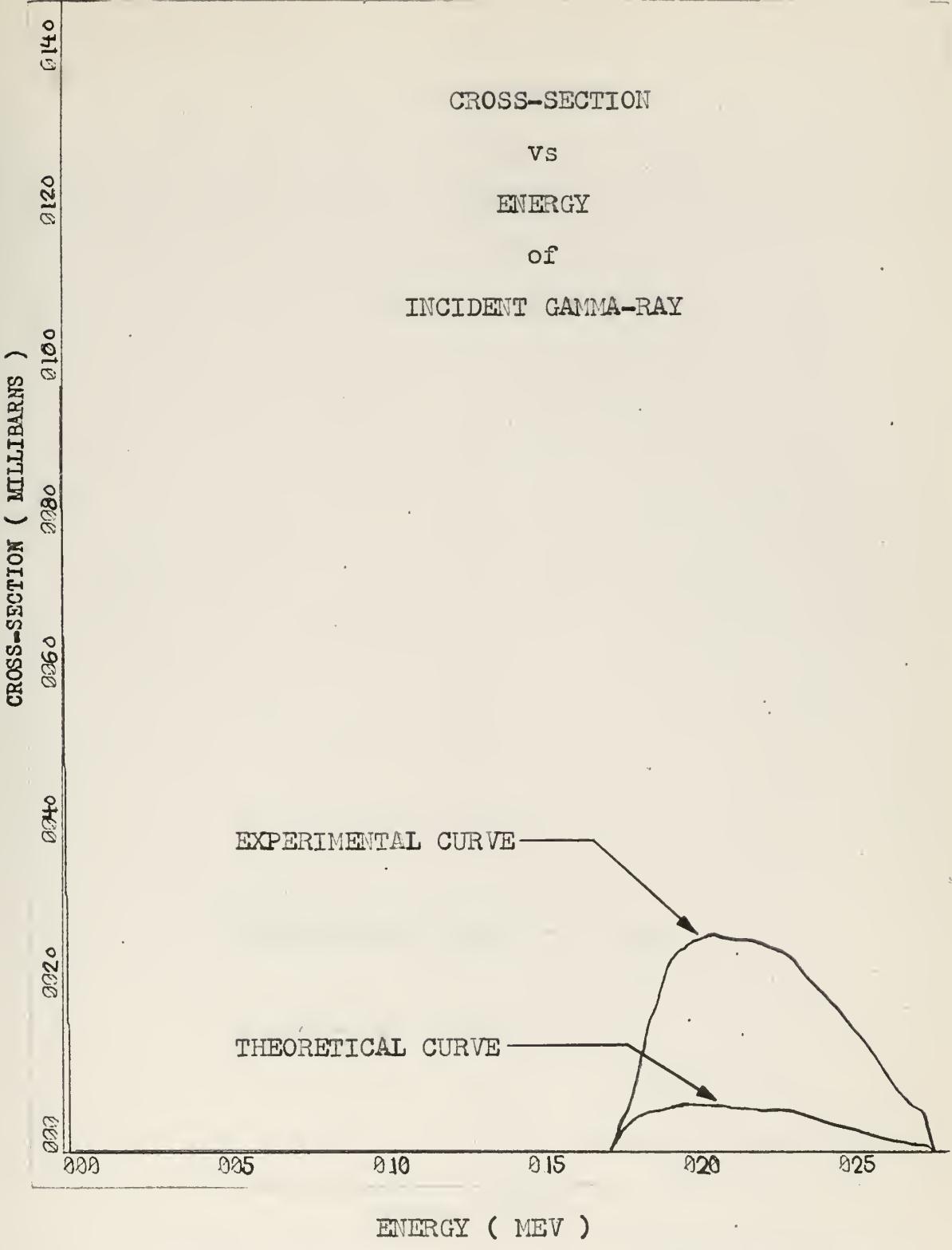
FIGURE 7

CROSS-SECTION
VS
ENERGY
of
INCIDENT GAMMA-RAY



DENSITY FUNCTION $P(E) = C(Z - E) \exp(A(E^n - Z^n))$
 $A = 0.001 \text{ MEV}^{-1}$

FIGURE 8

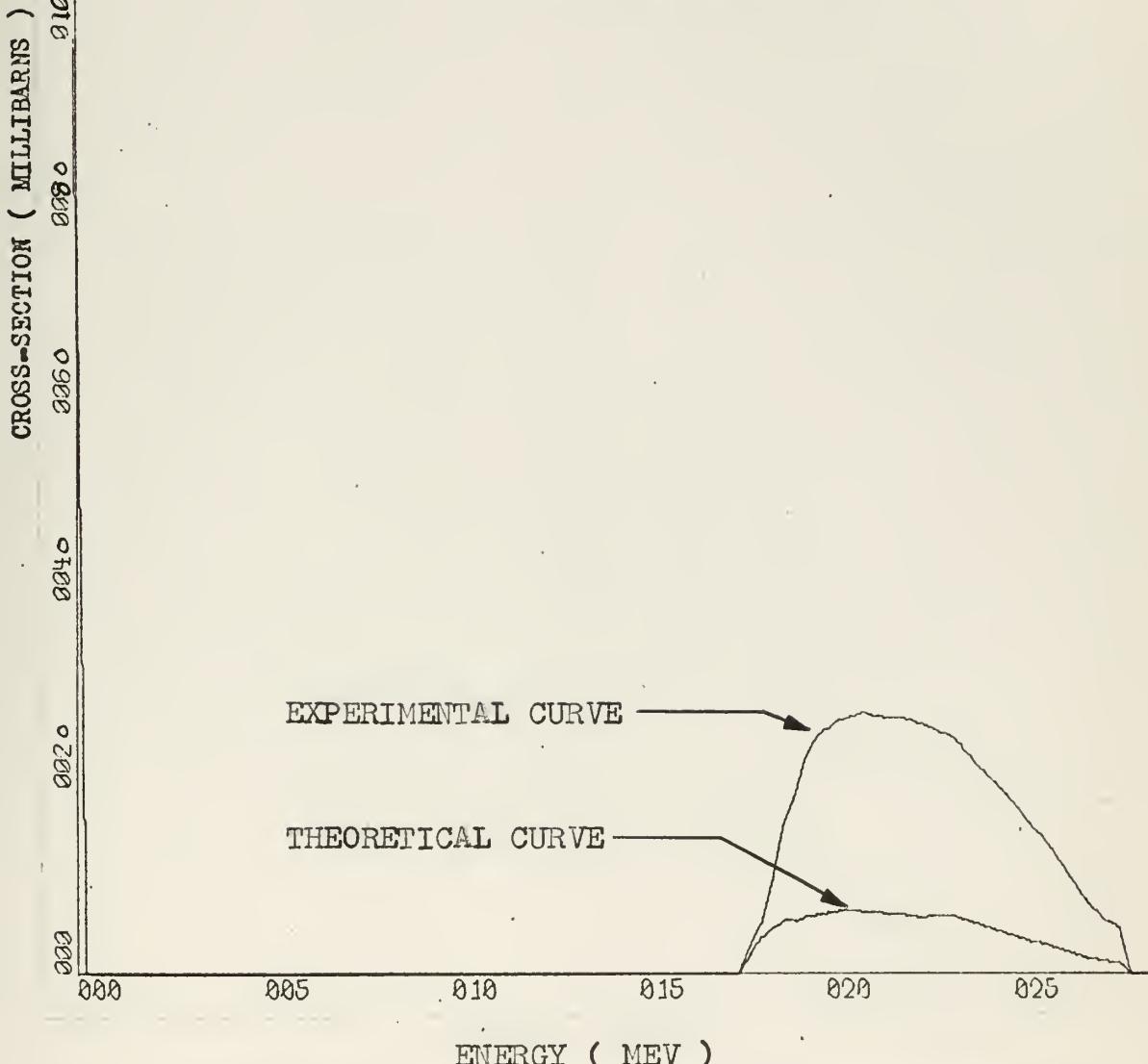


$$\text{DENSITY FUNCTION } P(E) = C(Z - E) \exp(A(E^n - Z^n))$$

$$A = 0.01 \text{ MEV}^{-1}$$

FIGURE 9

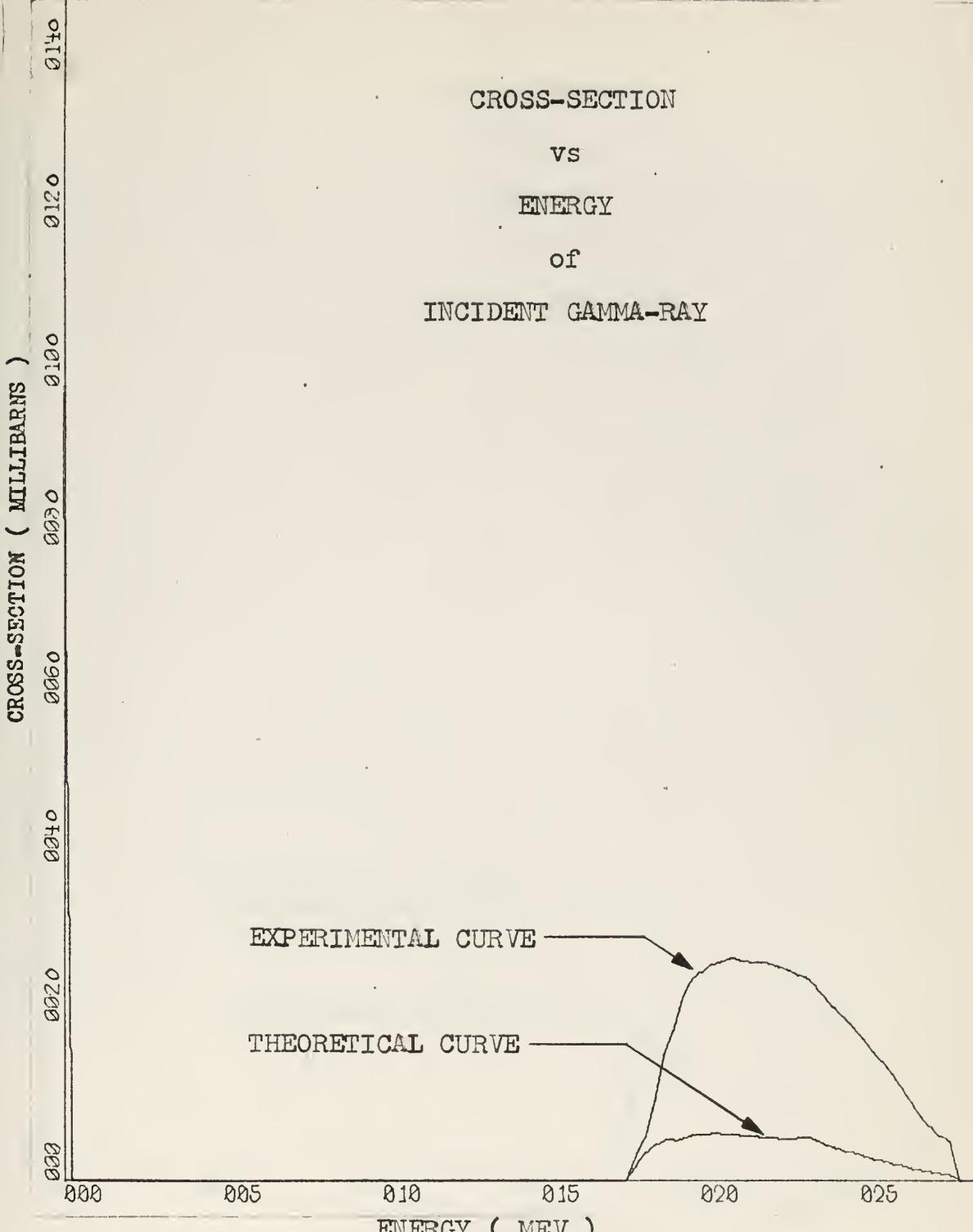
CROSS-SECTION
vs
ENERGY
of
INCIDENT GAMMA-RAY



DENSITY FUNCTION $P(E) = C(Z - E) \exp(A(E^n - Z^n))$
 $A = 0.1 \text{ MEV}^{-1}$

FIGURE 10

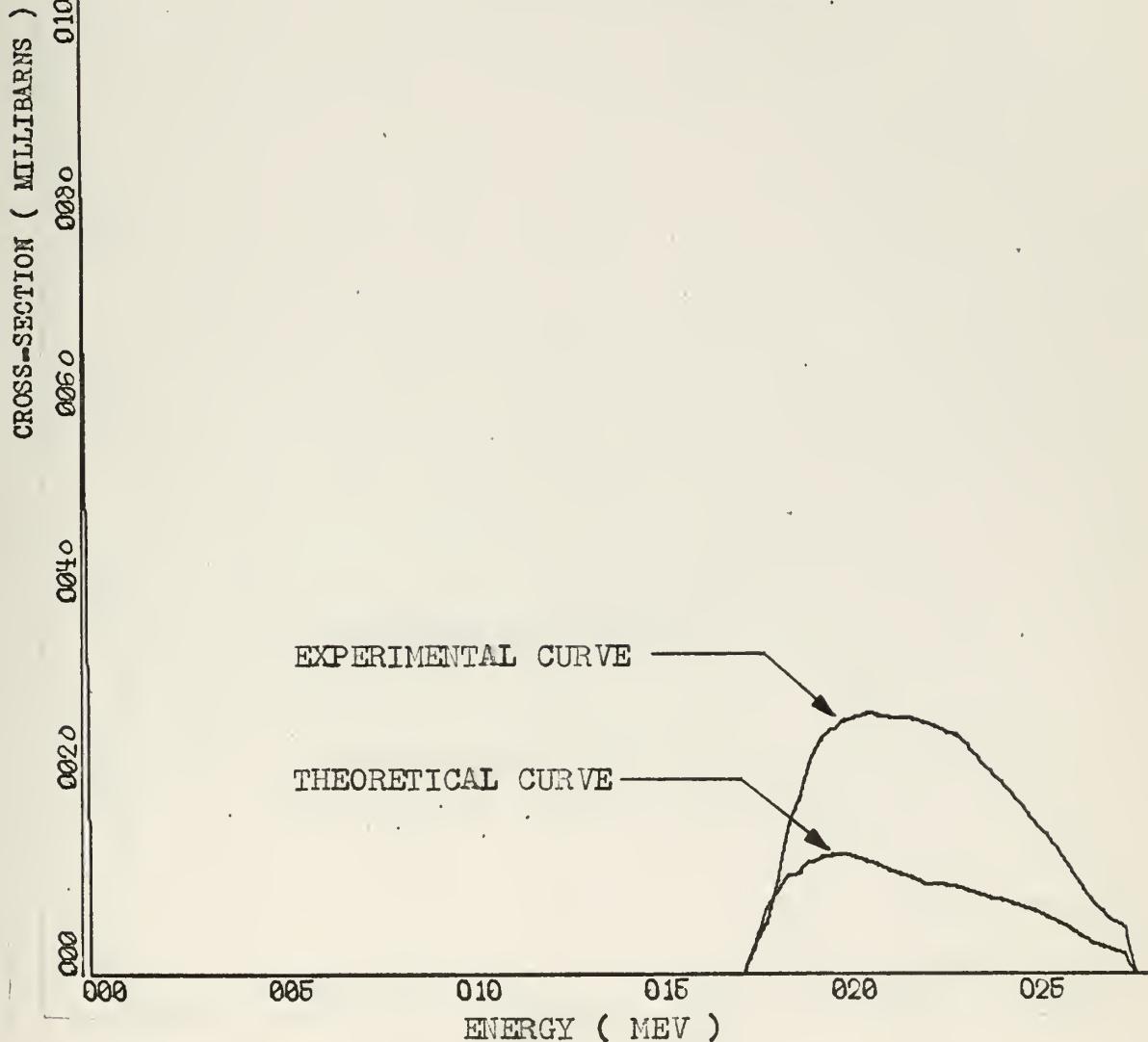
CROSS-SECTION
vs
ENERGY
of
INCIDENT GAMMA-RAY



DENSITY FUNCTION $P(E) = C(Z - E) \exp(A(E^n - Z^n))$
 $A = 0.5 \text{ MEV}^{-1}$

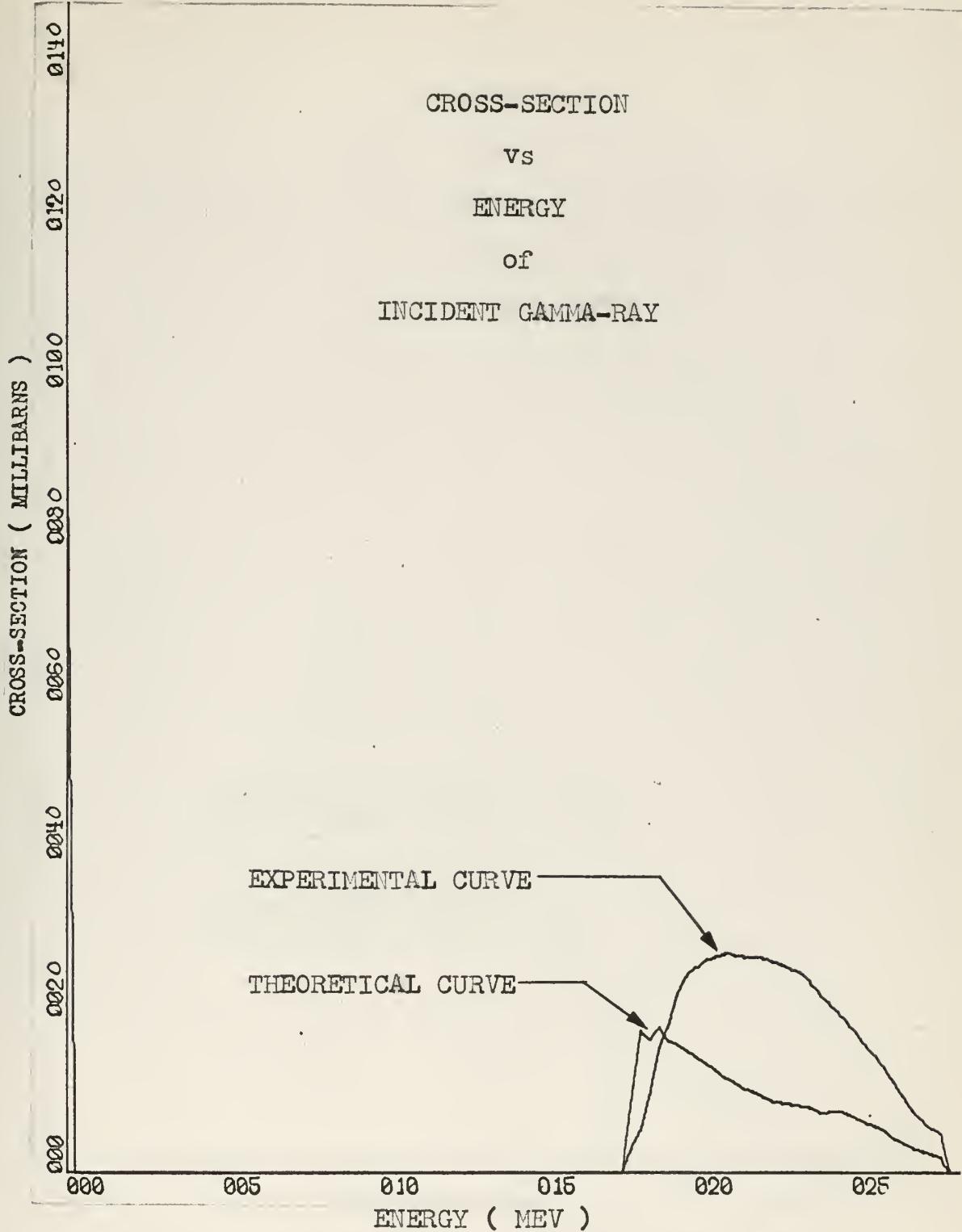
FIGURE 11

CROSS-SECTION
vs
ENERGY
of
INCIDENT GAMMA-RAY



DENSITY FUNCTION $P(E) = C(Z - E) \exp(A(E^n - Z^n))$
 $A = 1.0 \text{ MEV}^{-1}$

FIGURE 12

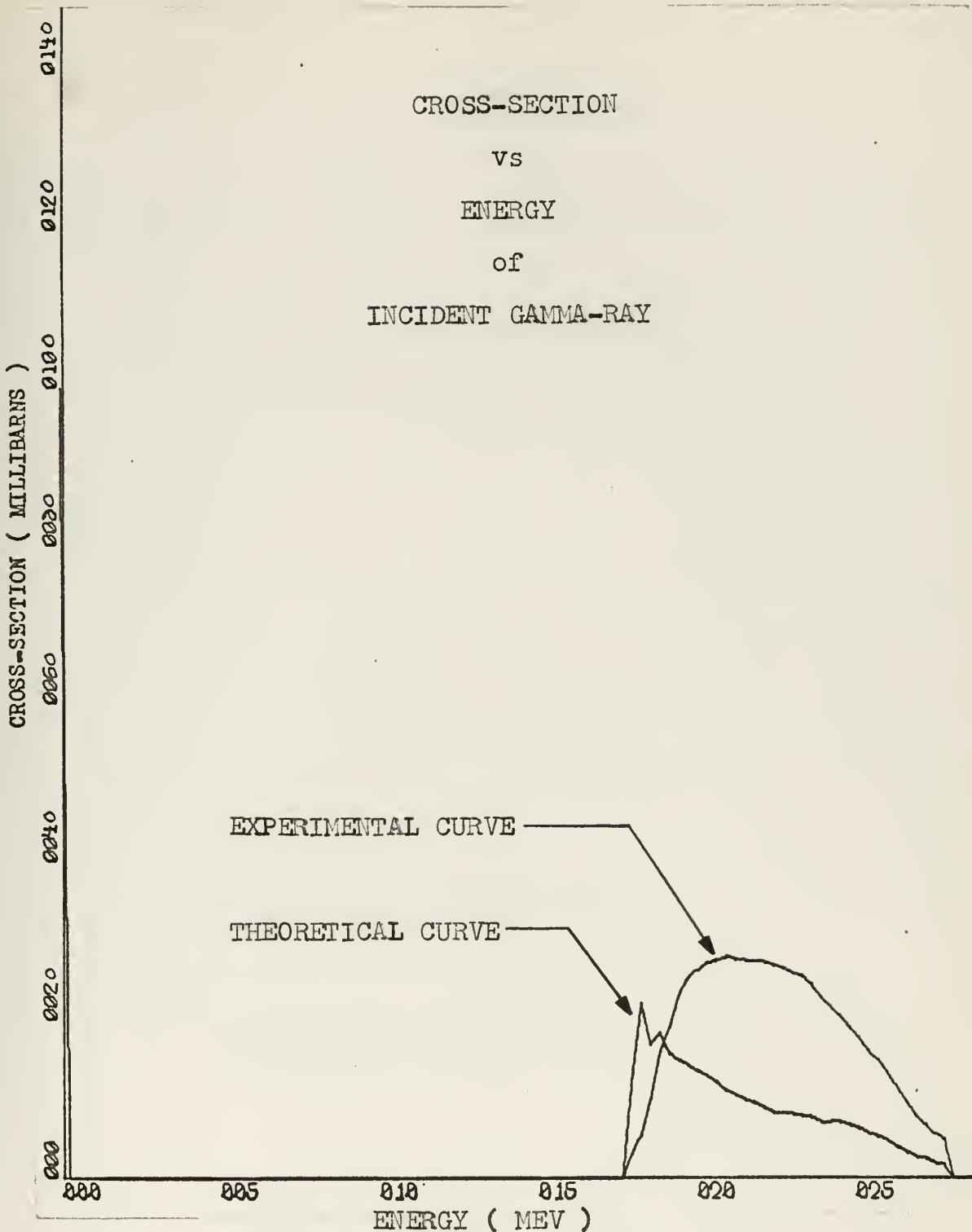


$$\text{DENSITY FUNCTION } P(E) = C(Z - E) \exp(A(E^n - Z^n))$$

$$A = 5.0 \text{ MEV}^{-1}$$

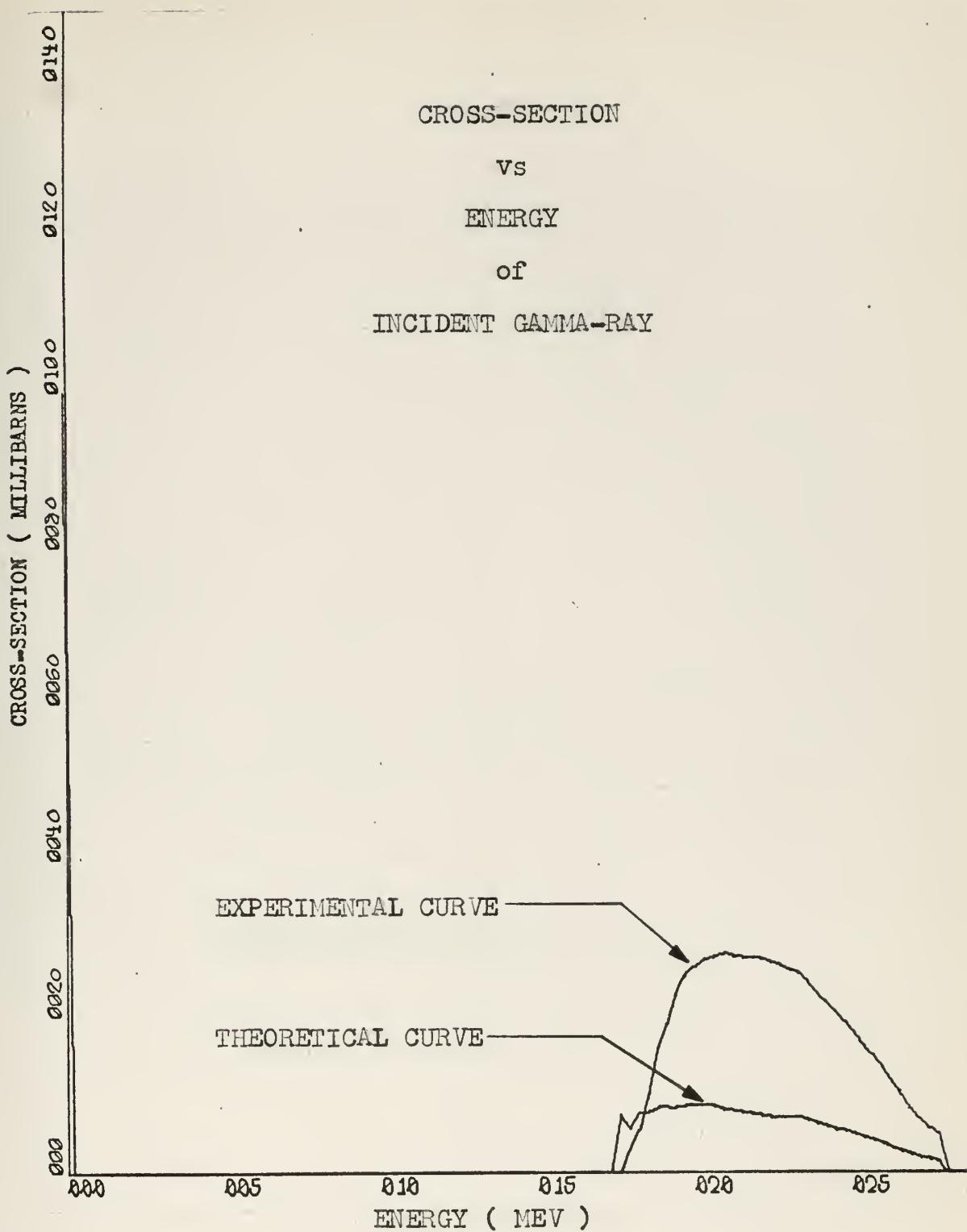
FIGURE 13

CROSS-SECTION
vs
ENERGY
of
INCIDENT GAMMA-RAY



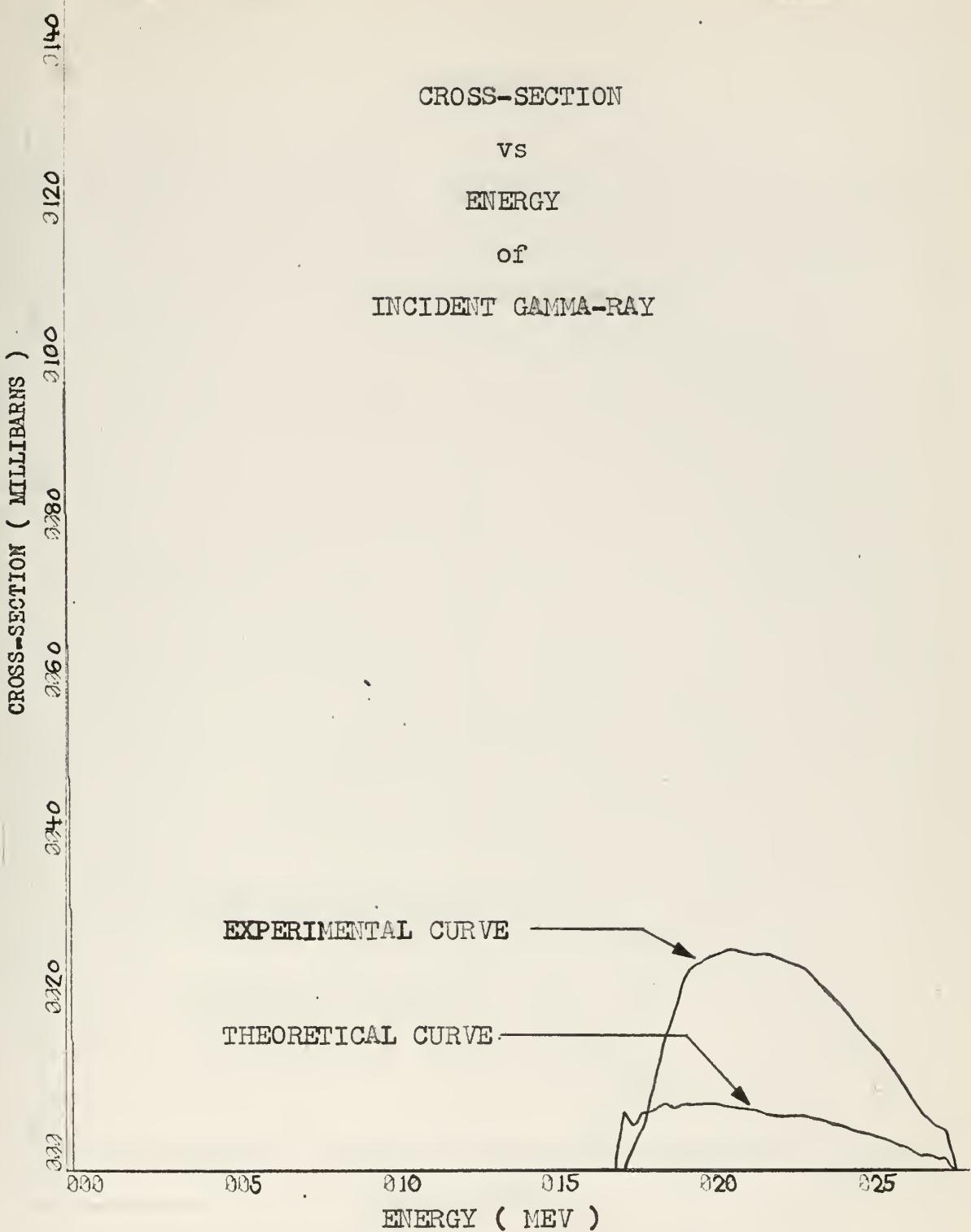
$$\text{DENSITY FUNCTION } P(E) = C(Z - E) \exp(A(E^n - Z^n))$$
$$A = 10.0 \text{ MEV}^{-1}$$

FIGURE 14



DENSITY FUNCTION $P(E) = C \exp(A E^n)$
 $A = 0.0005 \text{ MEV}^{-1}$

FIGURE 15



$$\begin{aligned}
 \text{DENSITY FUNCTION} \quad P(E) &\equiv C \exp(-AE^B) \\
 A &\equiv 0.01 \text{ MEV}^{-1}
 \end{aligned}$$

FIGURE 16

CROSS-SECTION
vs
ENERGY
of
INCIDENT GAMMA-RAY

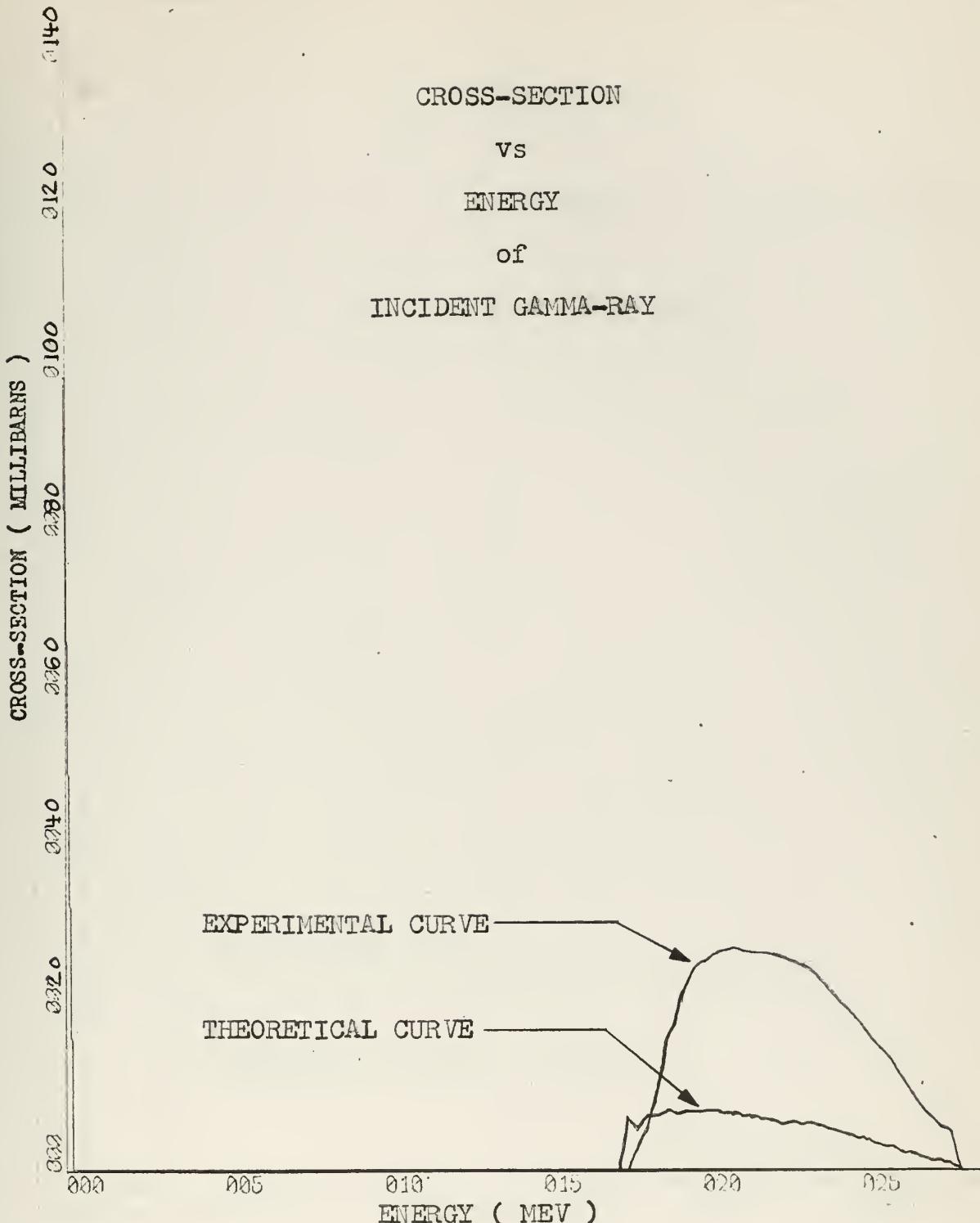
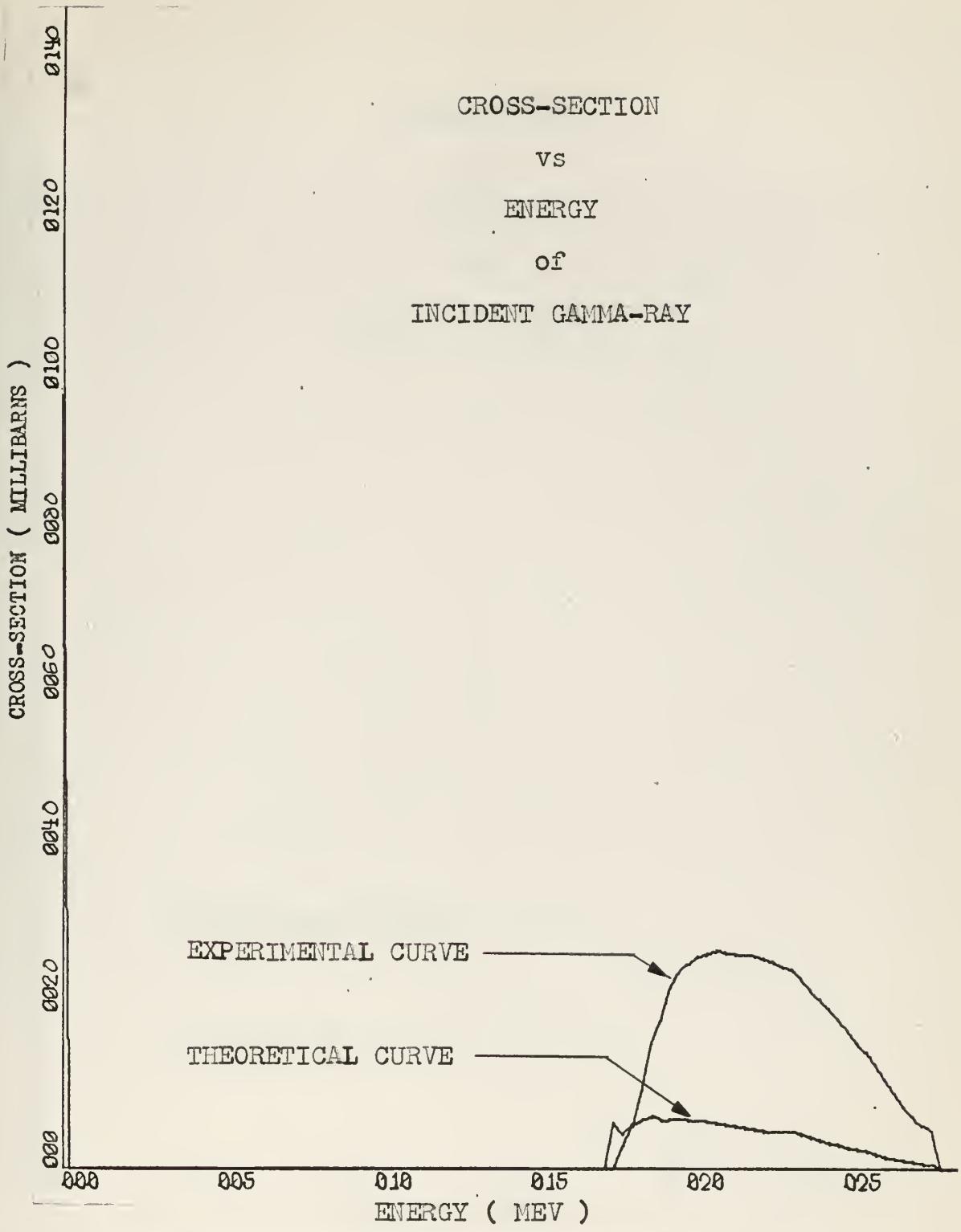


FIGURE 17

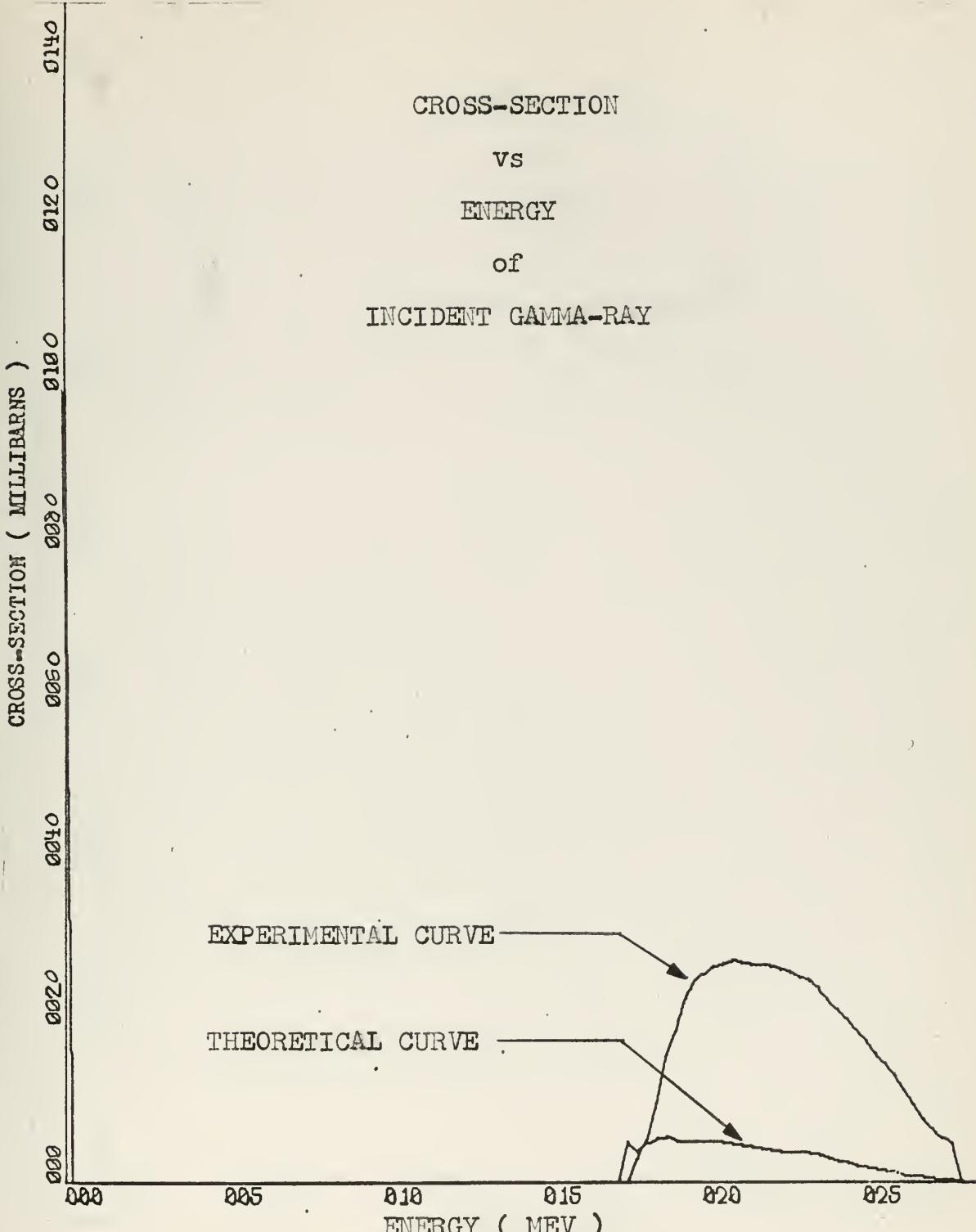


$$\text{DENSITY FUNCTION } P(E) = C \exp(-A E^n)$$

$$A = 0.2 \text{ MEV}^{-1}$$

FIGURE 18

CROSS-SECTION
vs
ENERGY
of
INCIDENT GAMMA-RAY



$$\text{DENSITY FUNCTION } P(E) = C \exp(-A E^n)$$
$$A = 0.3 \text{ MEV}^{-1}$$

FIGURE 19

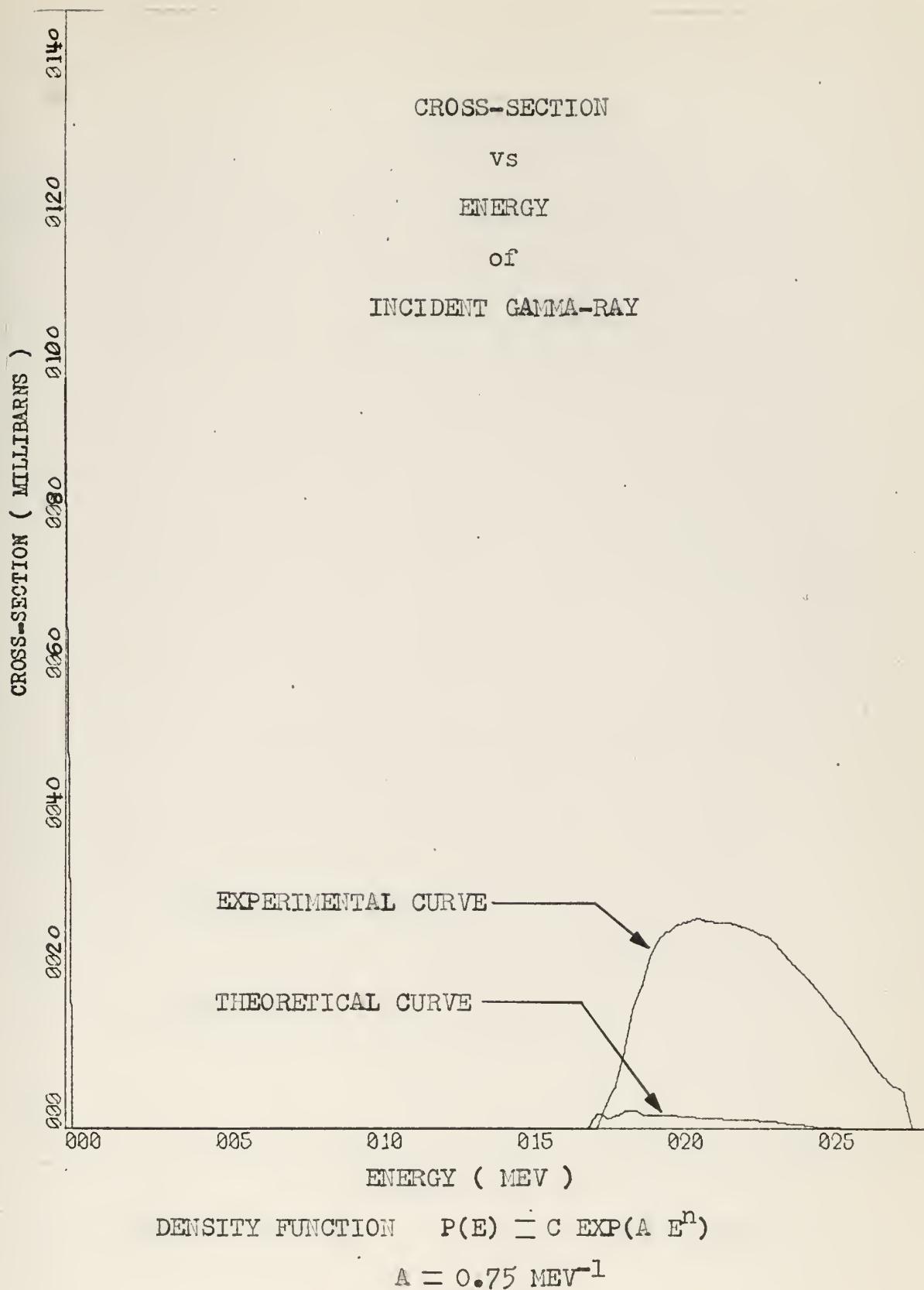
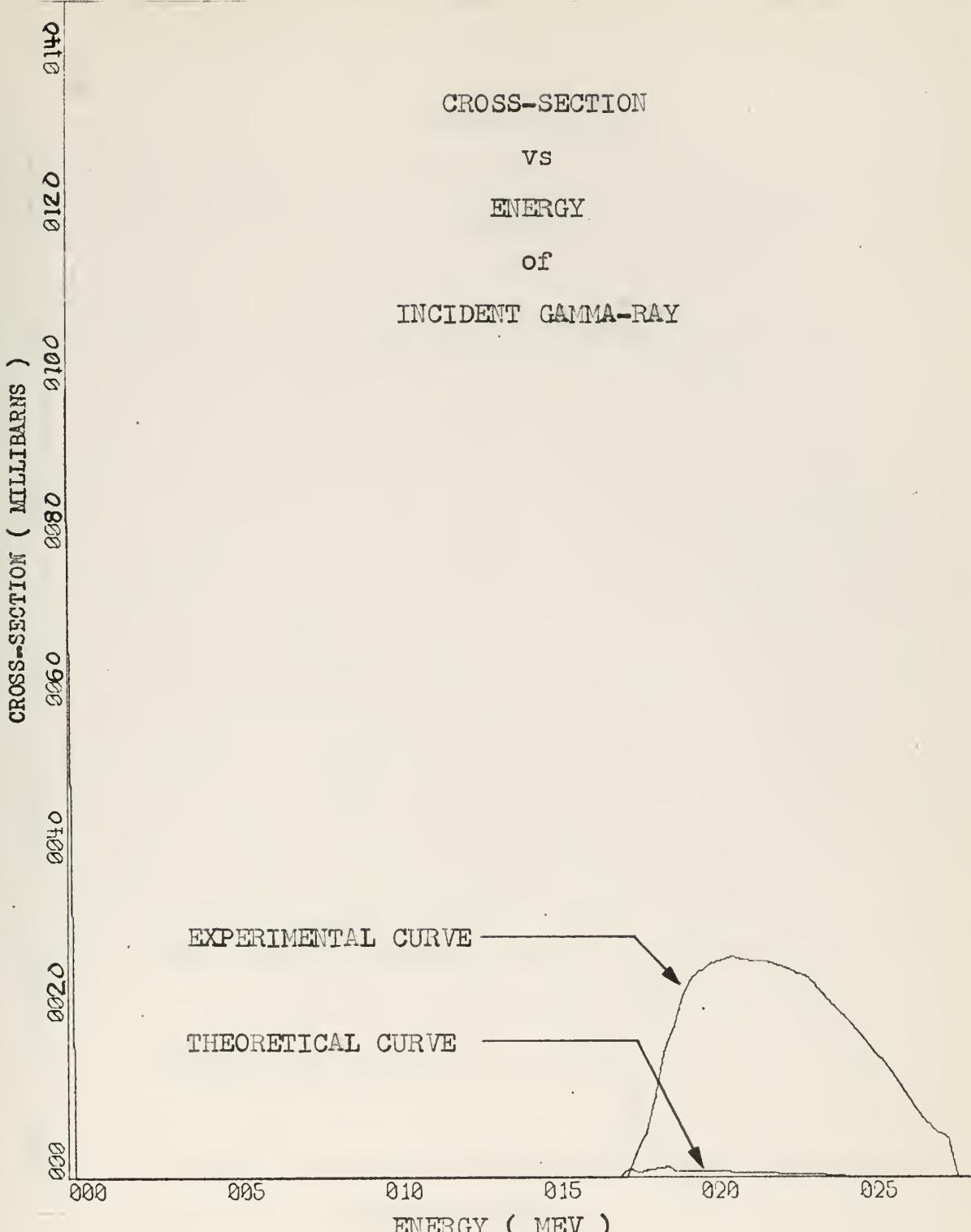


FIGURE 20



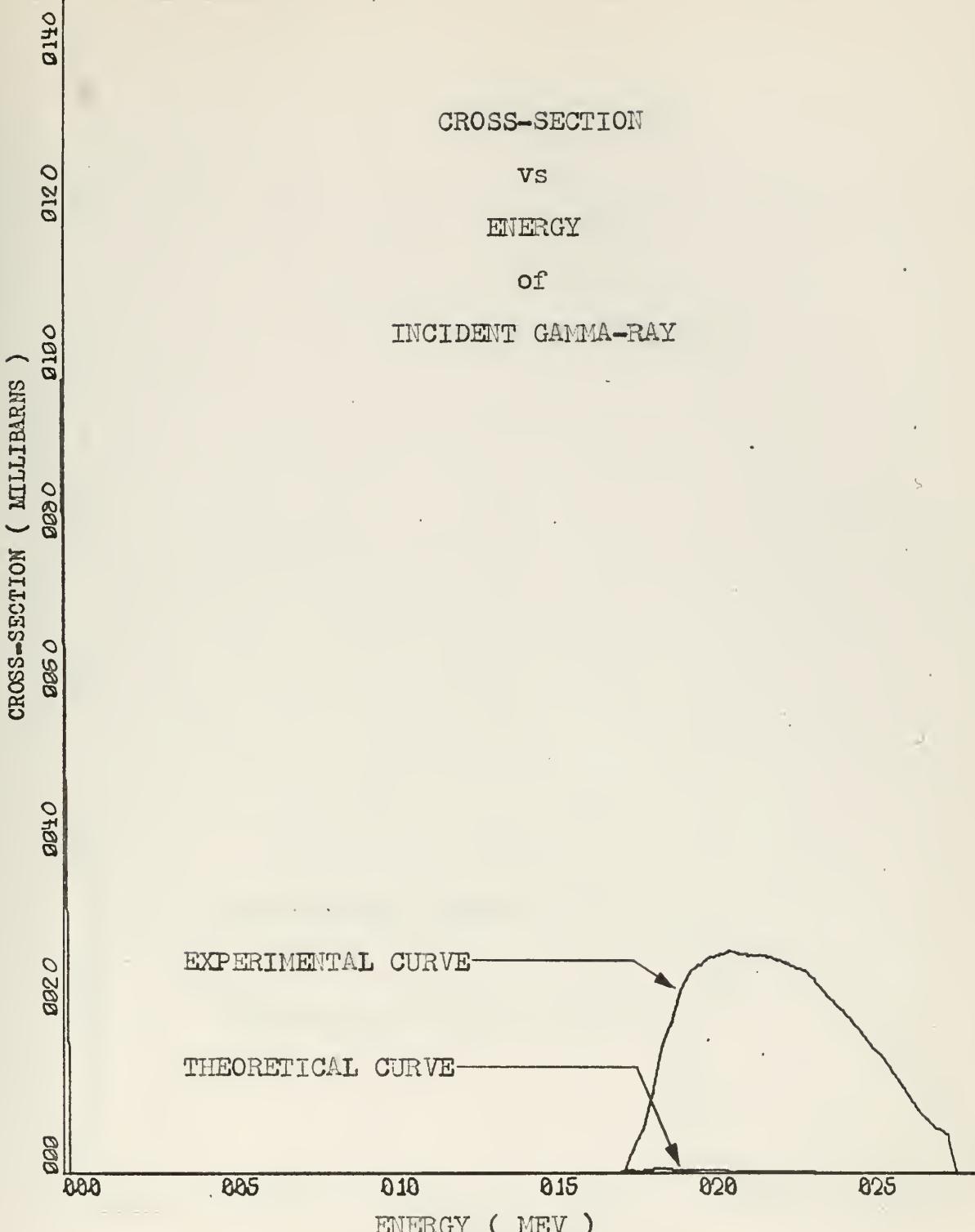
DENSITY FUNCTION

$$P(E) = C \exp(-A E^n)$$

$$A = 1.0 \text{ MEV}^{-1}$$

FIGURE 21

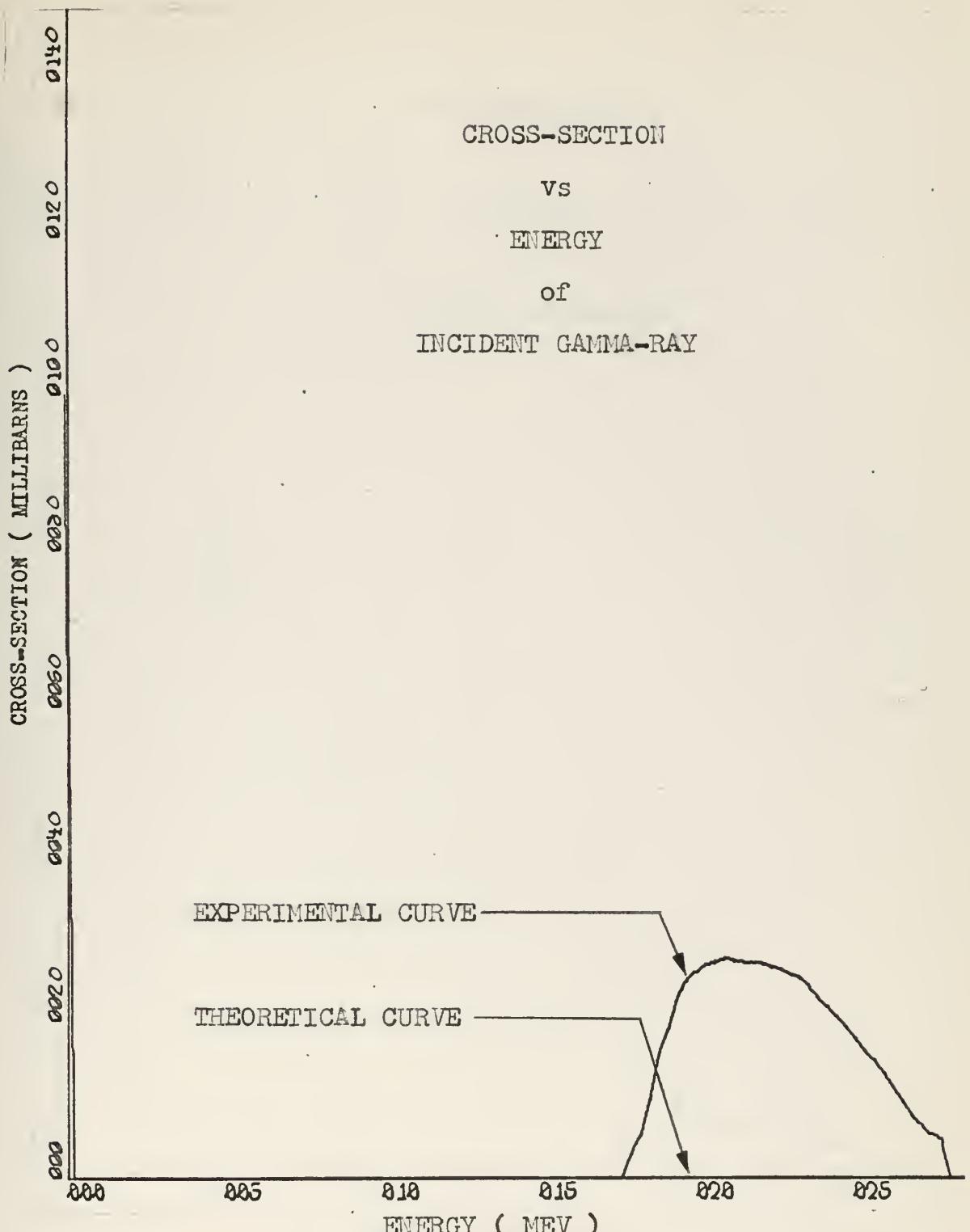
CROSS-SECTION
vs
ENERGY
of
INCIDENT GAMMA-RAY



DENSITY FUNCTION $P(E) = C \exp(-A E^n)$

$$A = 1.25 \text{ MEV}^{-1}$$

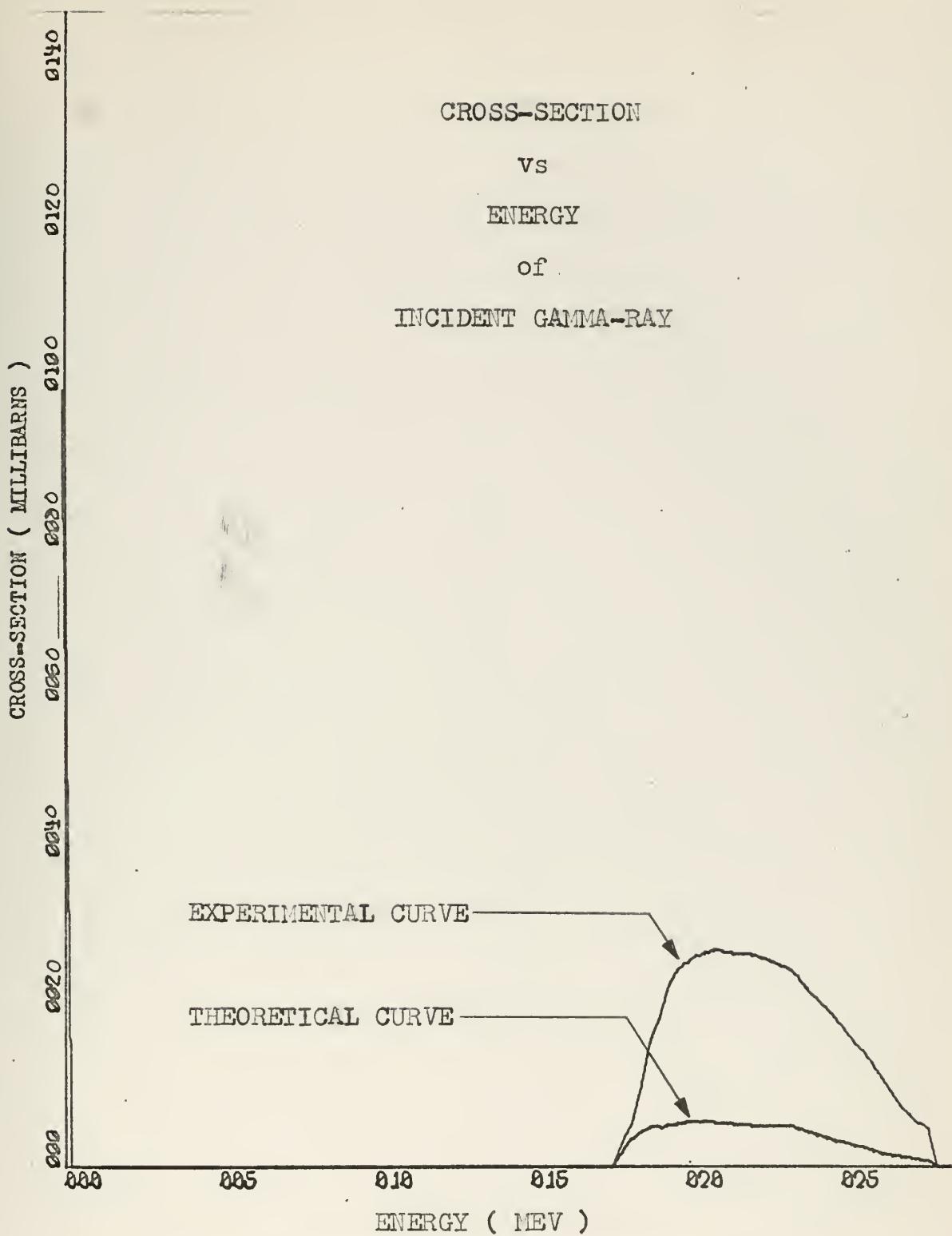
FIGURE 22



$$\text{DENSITY FUNCTION } P(E) = C \exp(-A E^n)$$

$$A = 10.0 \text{ MeV}^{-1}$$

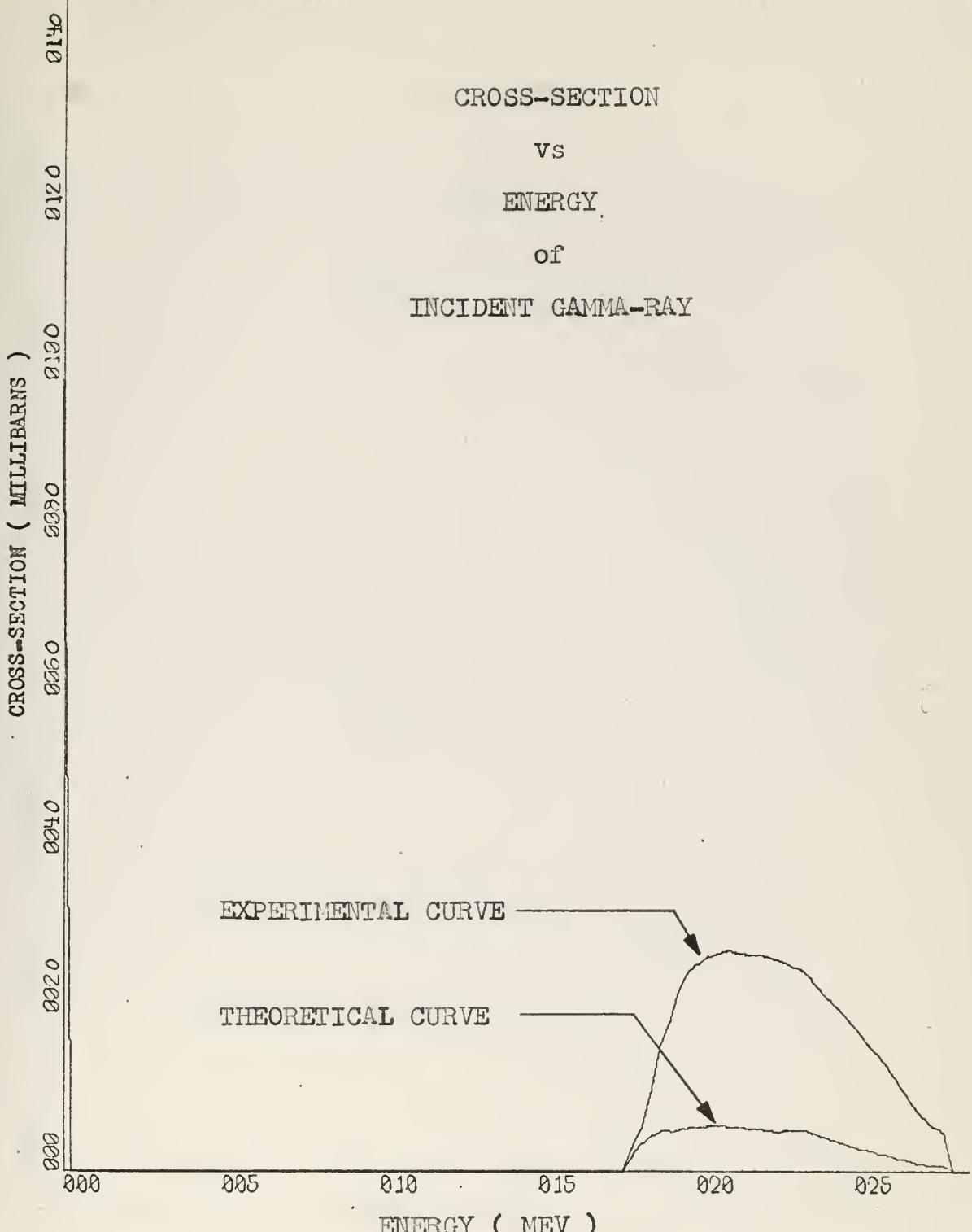
FIGURE 23



$$\text{DENSITY FUNCTION } P(E) = C Z \exp(-A E^n)$$

$$A = 0.0001 \text{ MEV}^{-1}$$

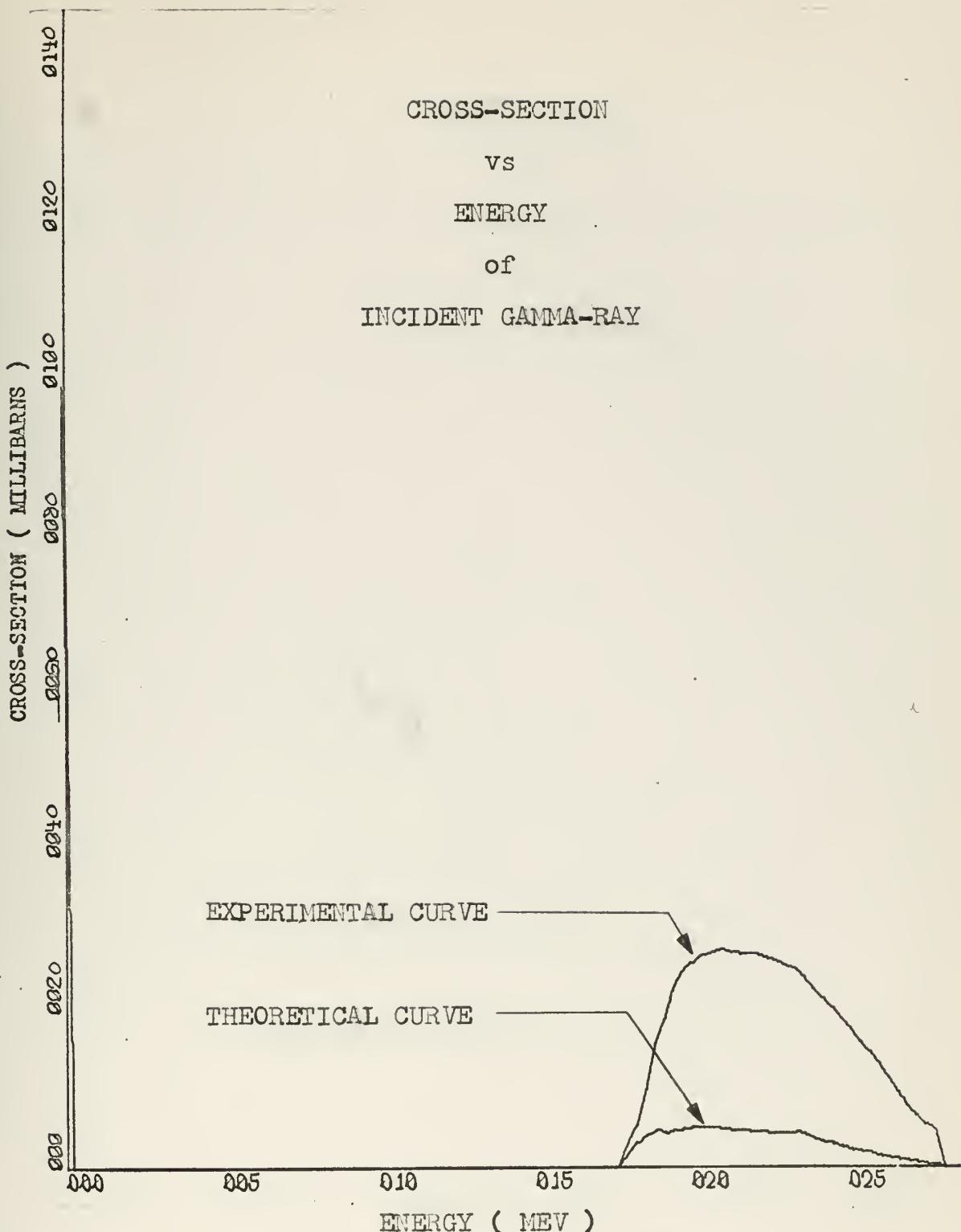
FIGURE 24



$$\text{DENSITY FUNCTION } P(E) = C Z \exp(A E^n)$$

$$A = 0.01 \text{ MEV}^{-1}$$

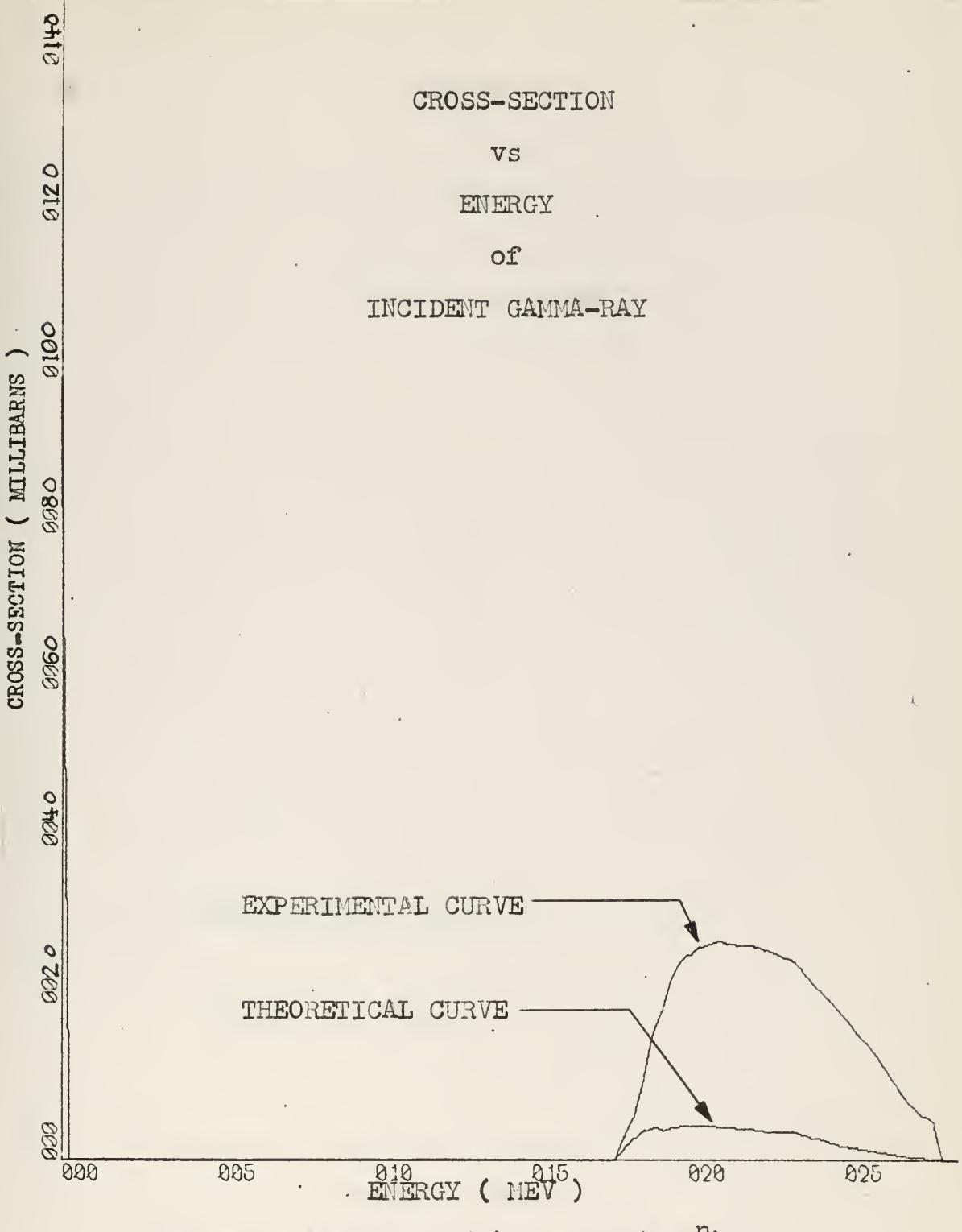
FIGURE 25



DENSITY FUNCTION $P(E) = C Z \exp(-A E^n)$

$$A = 0.1 \text{ MEV}^{-1}$$

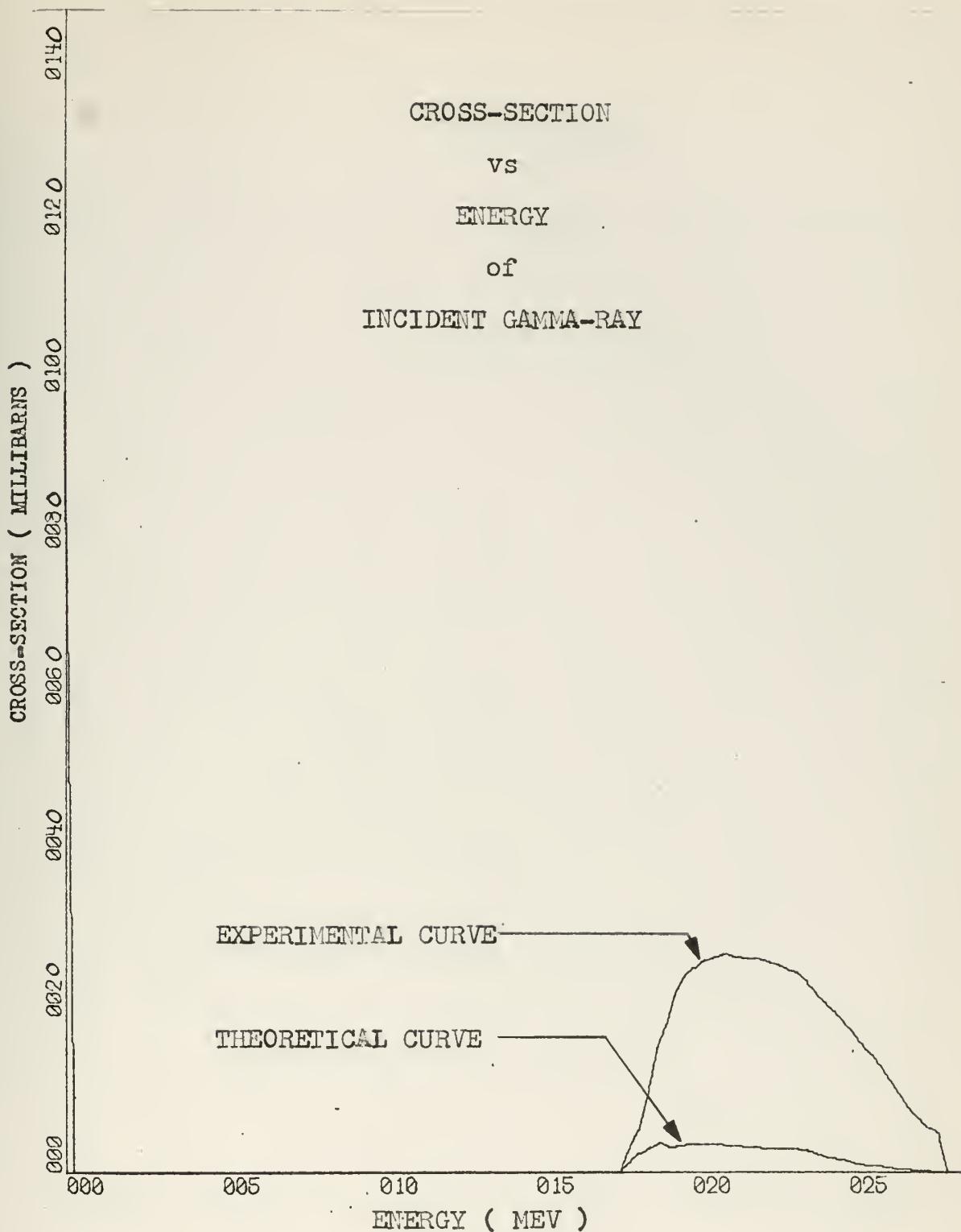
FIGURE 26



DENSITY FUNCTION $P(E) = C \cdot Z \exp(-A E^n)$

$$A = 0.2 \text{ MEV}^{-1}$$

FIGURE 27



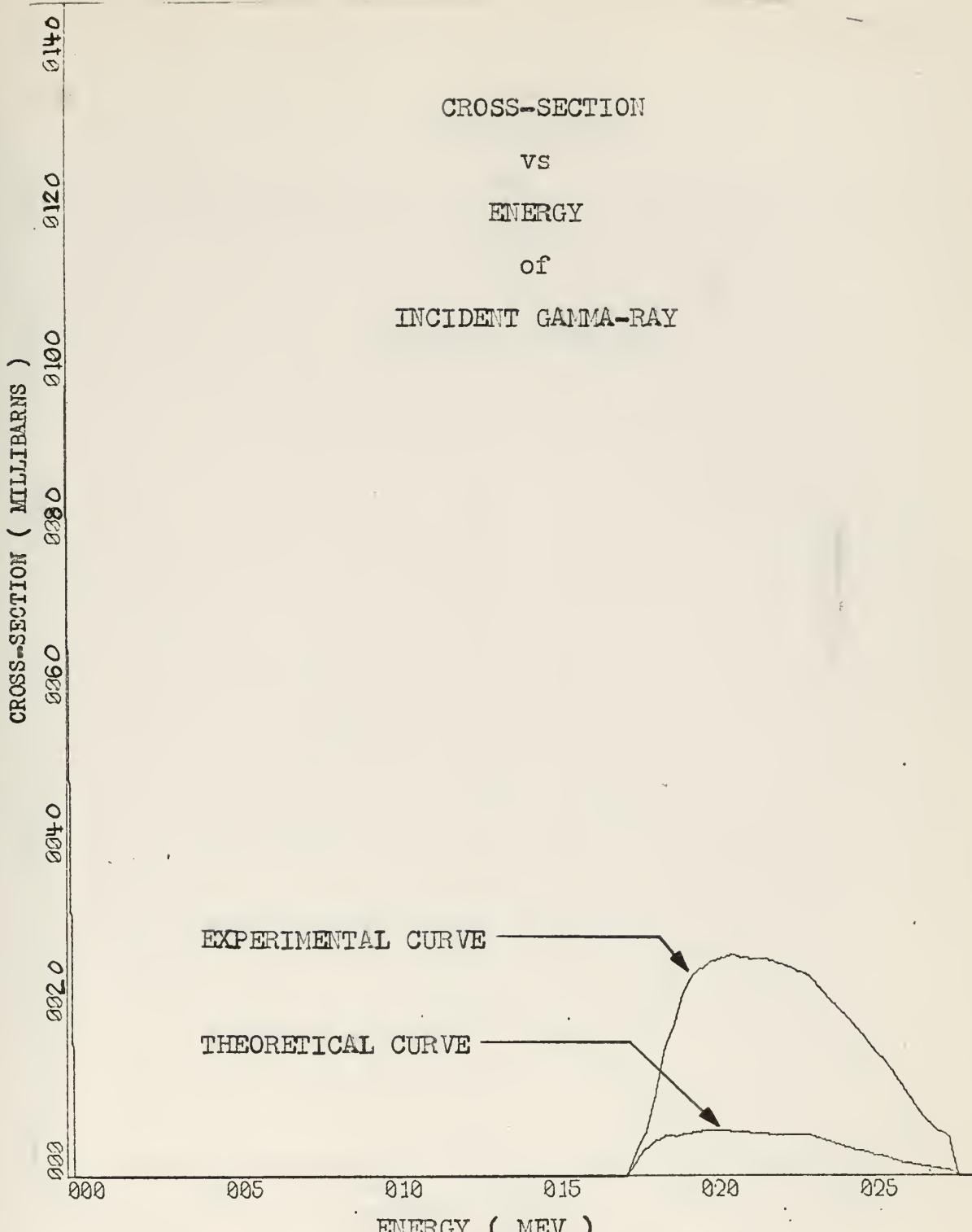
DENSITY FUNCTION $P(E) = C Z \exp(-A E^n)$

$$A = 0.3 \text{ MEV}^{-1}$$

FIGURE 28

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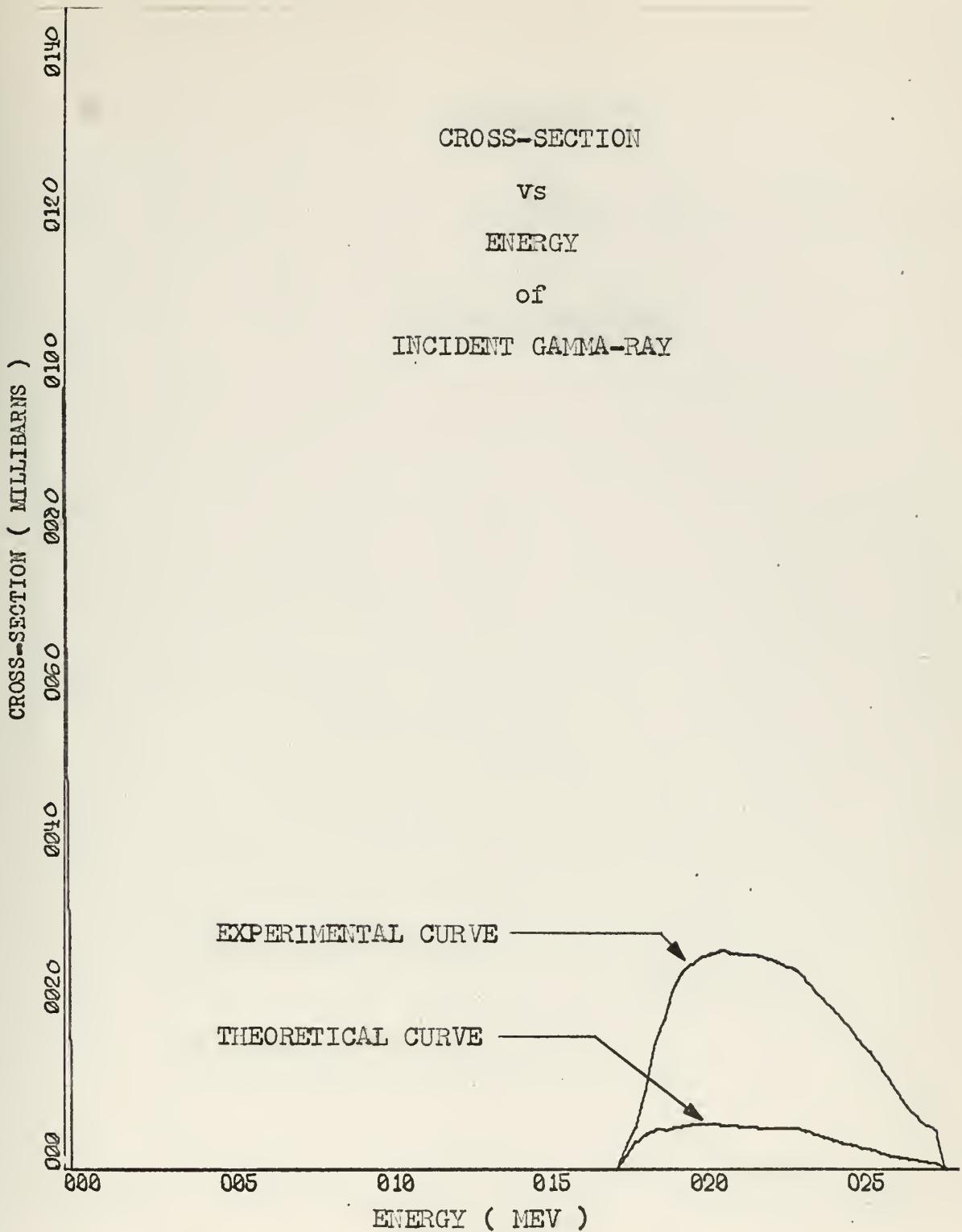
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DENSITY FUNCTION $P(E) = C Z \exp(-A E^n)$

$$A = 0.5 \text{ MEV}^{-1}$$

FIGURE 29



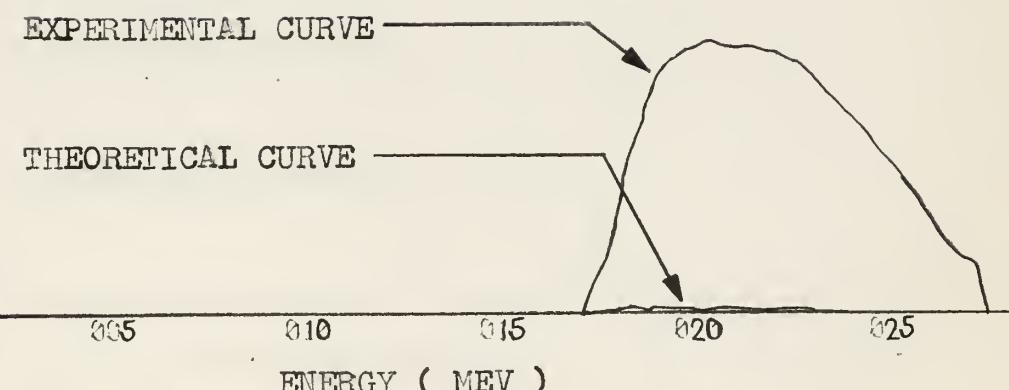
DENSITY FUNCTION $P(E) = C Z \exp(-A E^n)$

$$A = 0.8 \text{ MEV}^{-1}$$

FIGURE 30

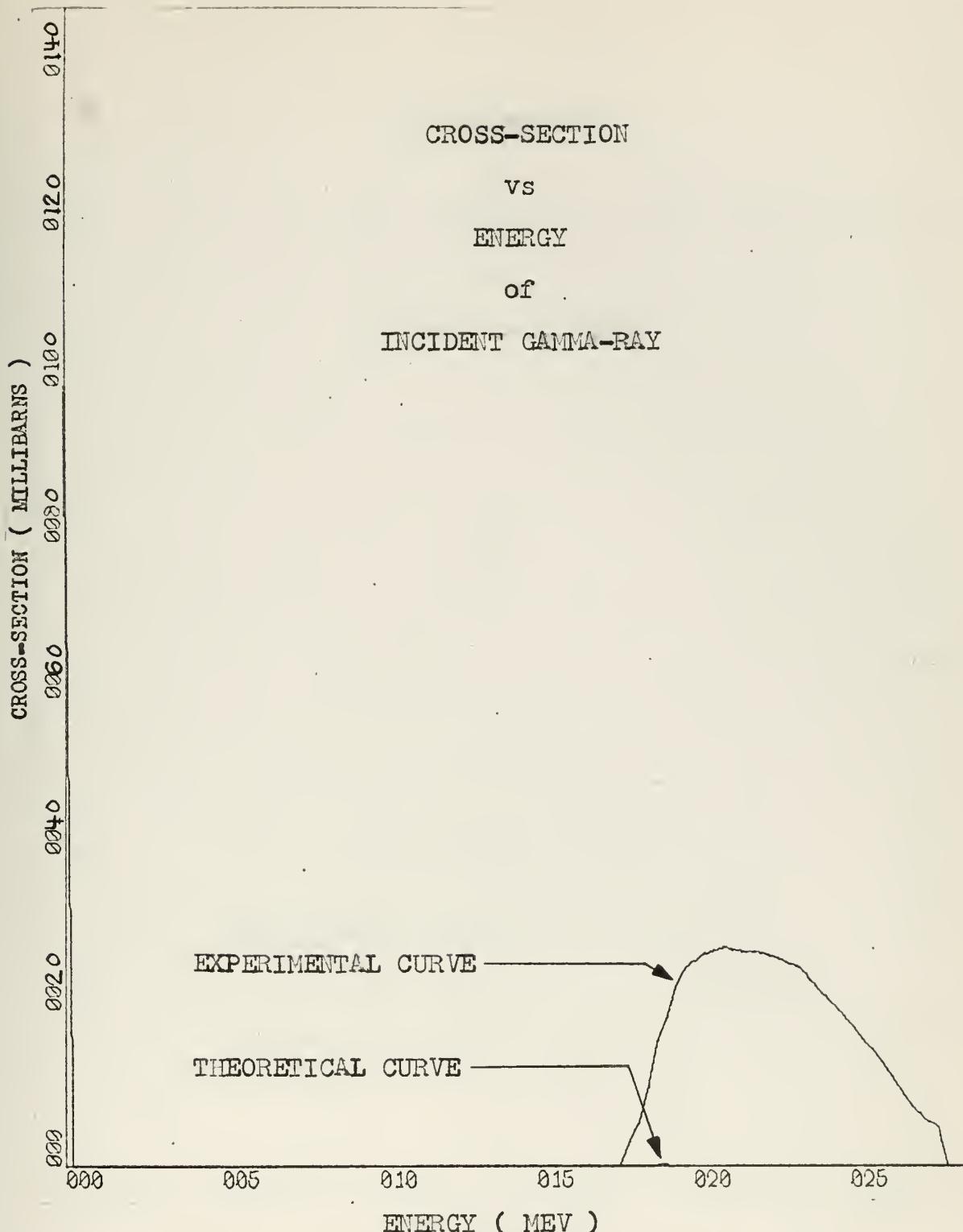
CROSS-SECTION (MILLIBARNS)
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CROSS-SECTION
vs
ENERGY
of
INCIDENT GAMMA-RAY



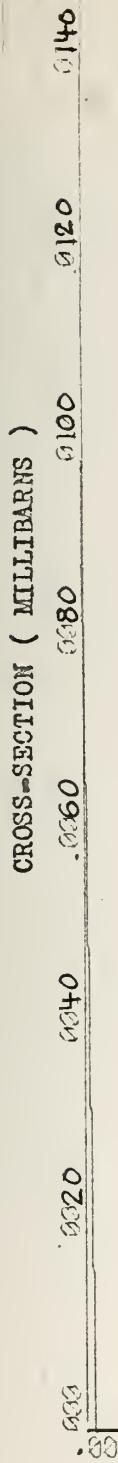
DENSITY FUNCTION $P(E) = C Z \exp(-A E^n)$
 $A = 1.0 \text{ MEV}^{-1}$

FIGURE 31



DENSITY FUNCTION $P(E) = C Z \exp(-A E^n)$
 $A = 1.5 \text{ MEV}^{-1}$

FIGURE 32

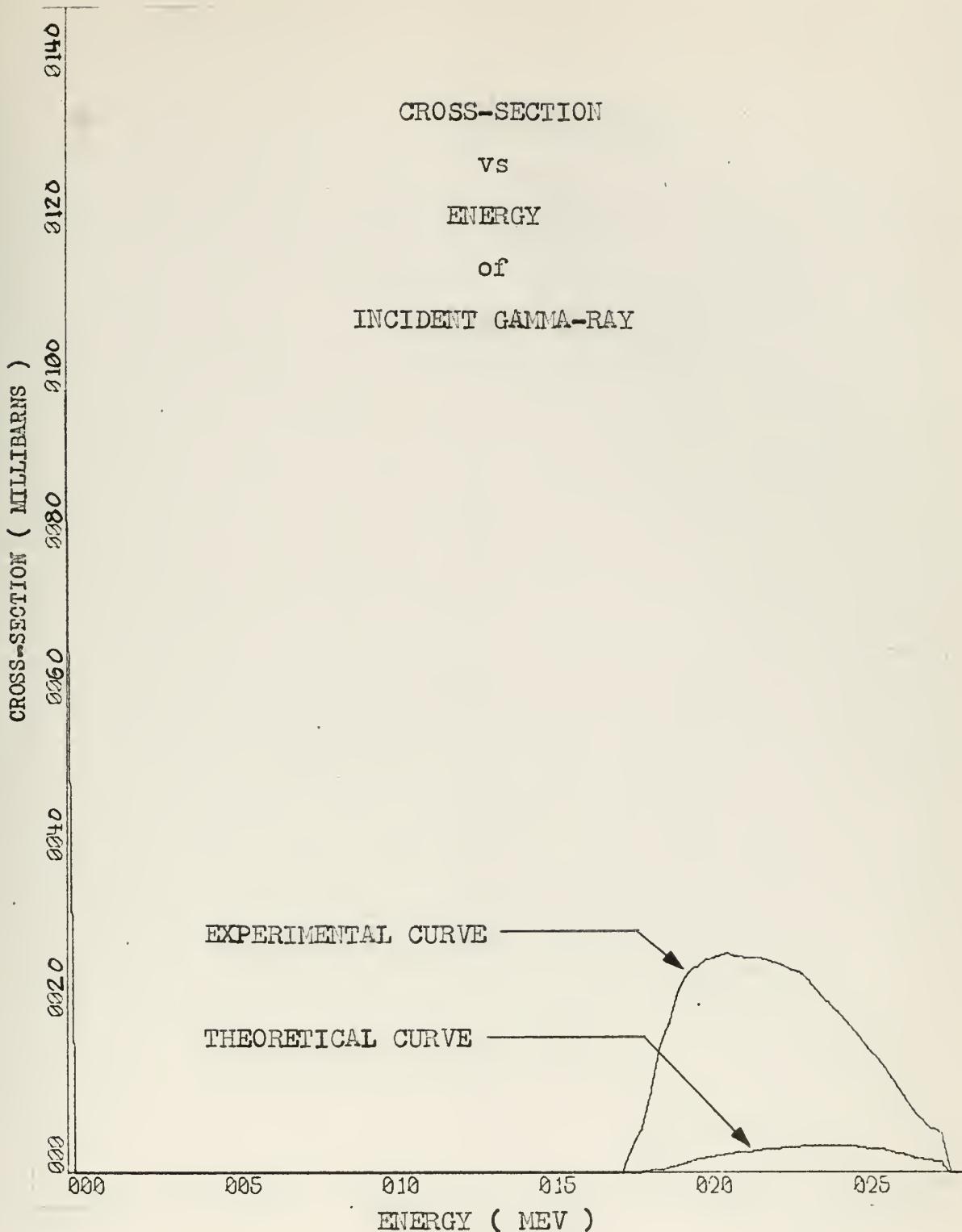


CROSS-SECTION
vs
ENERGY
of
INCIDENT GAMMA-RAY

DENSITY FUNCTION $P(E) = C Z \exp(-A E^n)$

$$A = 10.0 \text{ MEV}^{-1}$$

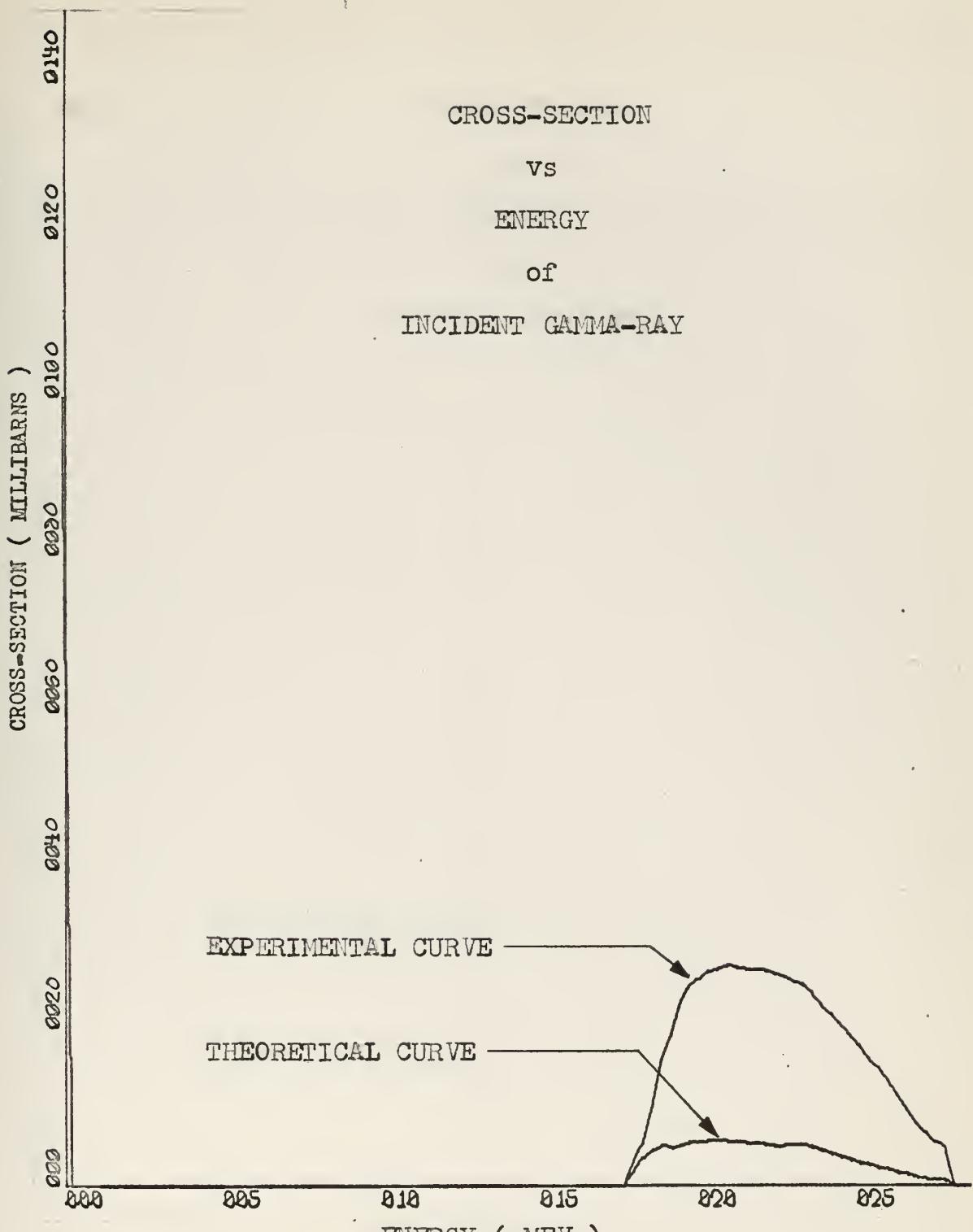
FIGURE 33



$$\text{DENSITY FUNCTION } P(E) = C E \exp(A E^n)$$

$$A = 0.0001 \text{ MEV}^{-1}$$

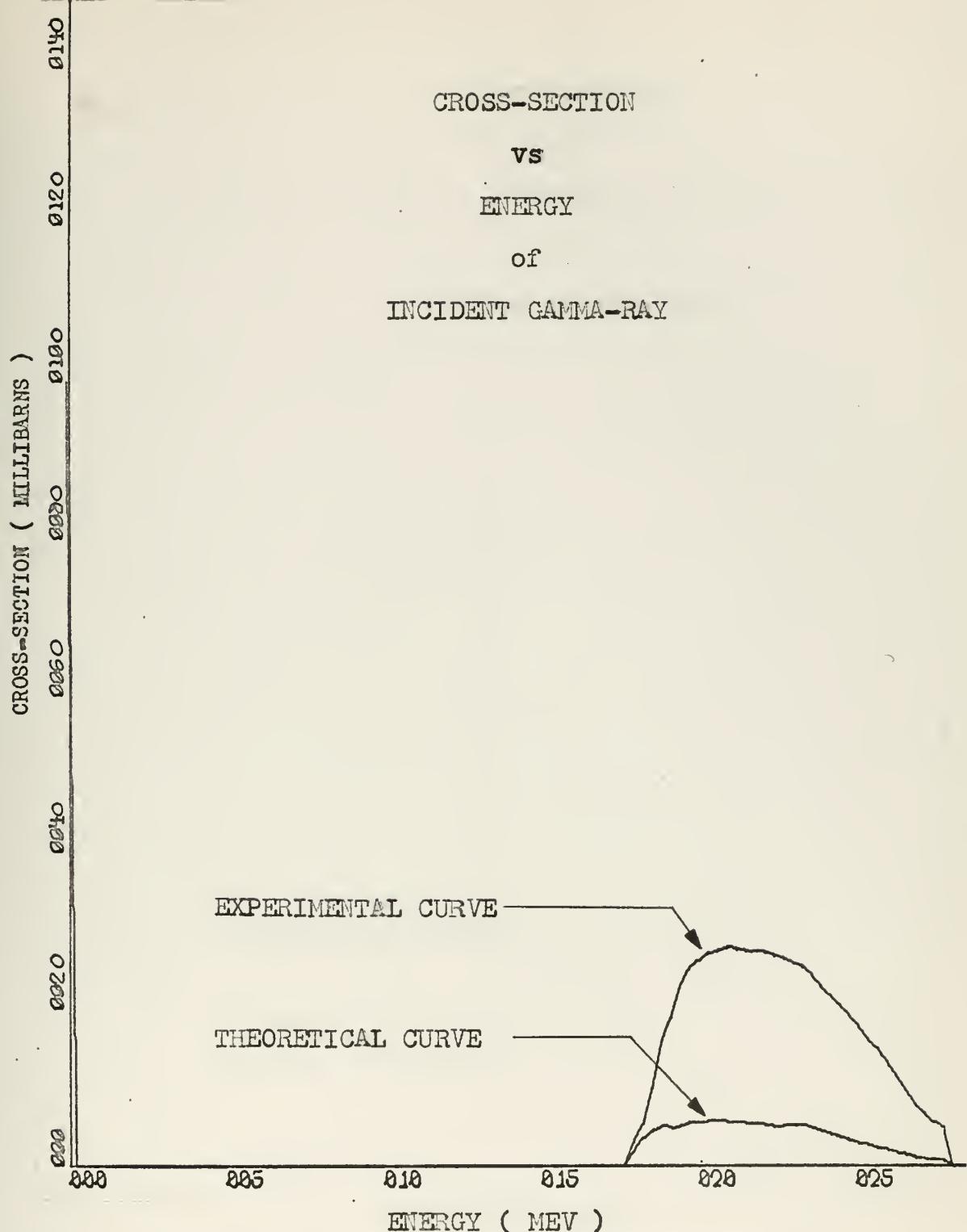
FIGURE 34



$$\text{DENSITY FUNCTION } P(E) = C E \exp(A E^n)$$

$$A = 0.0005 \text{ MEV}^{-1}$$

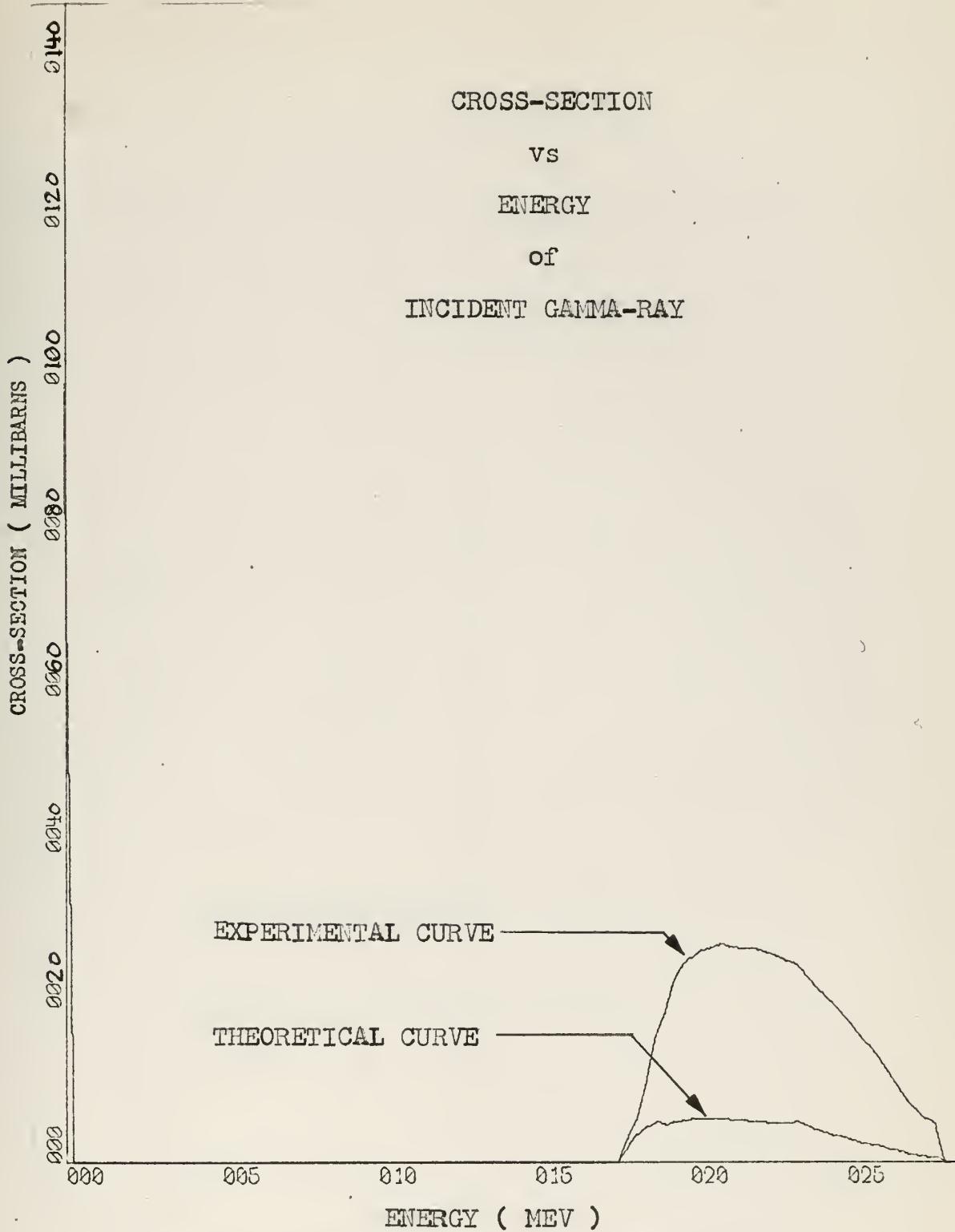
FIGURE 35



$$\text{DENSITY FUNCTION } P(E) = C E \exp(A E^n)$$

$$A = 0.001 \text{ MEV}^{-1}$$

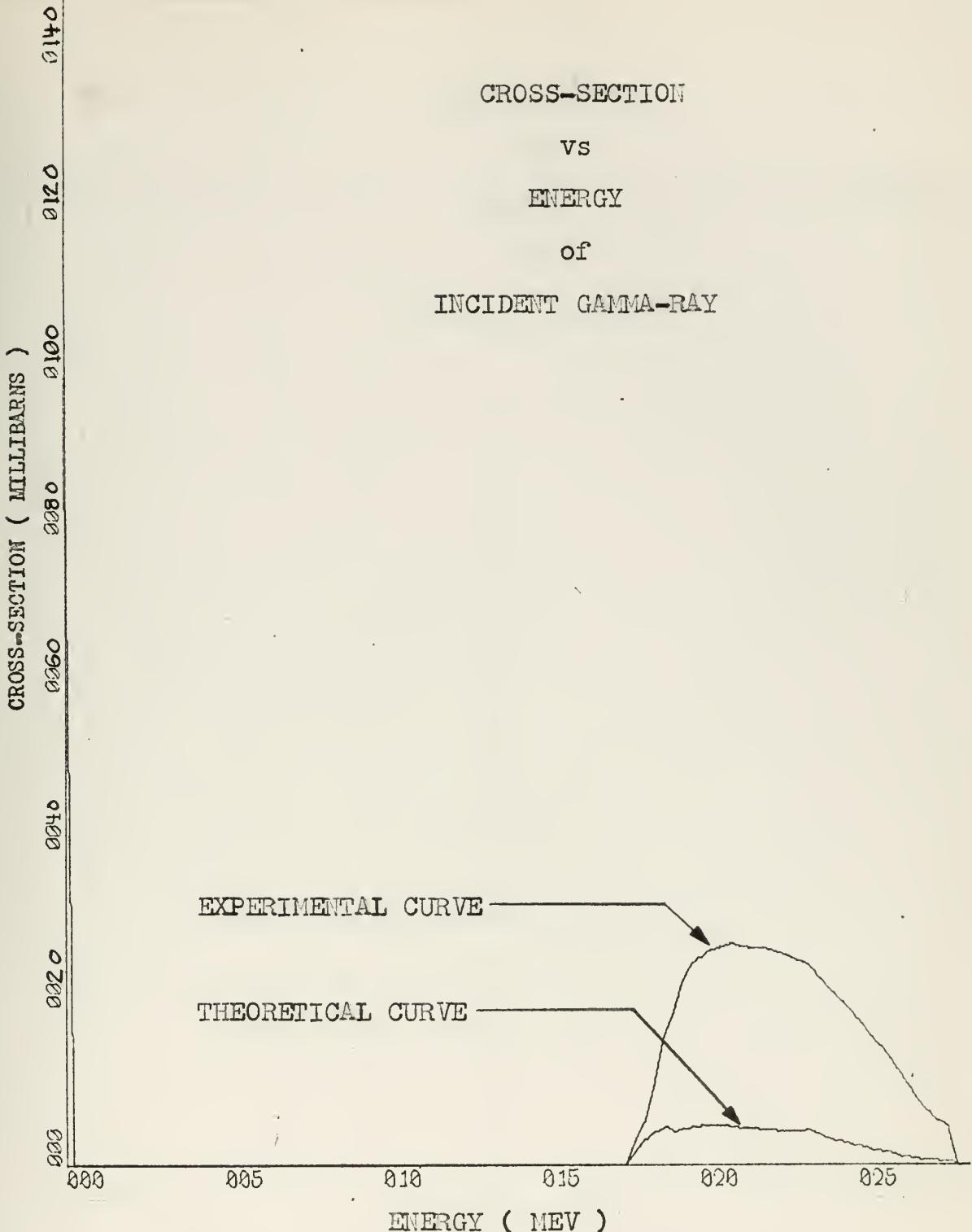
FIGURE 36



DENSITY FUNCTION $P(E) = C E \exp(-A E^n)$
 $A = 0.01 \text{ MEV}^{-1}$

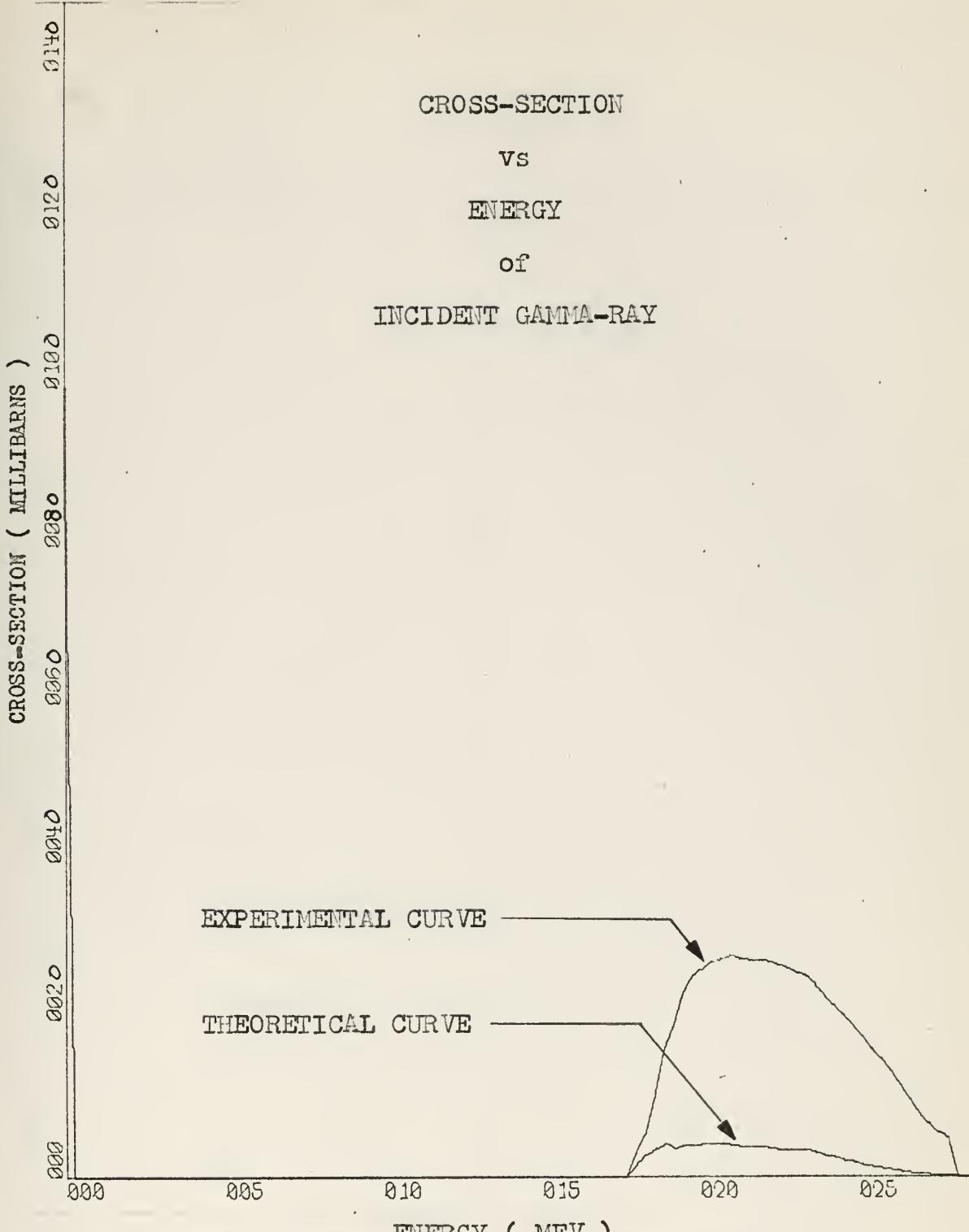
FIGURE 37

CROSS-SECTION
vs
ENERGY
of
INCIDENT GAMMA-RAY



DENSITY FUNCTION $P(E) = C E \exp(-A E^2)$
 $A = 0.1 \text{ MEV}^{-1}$

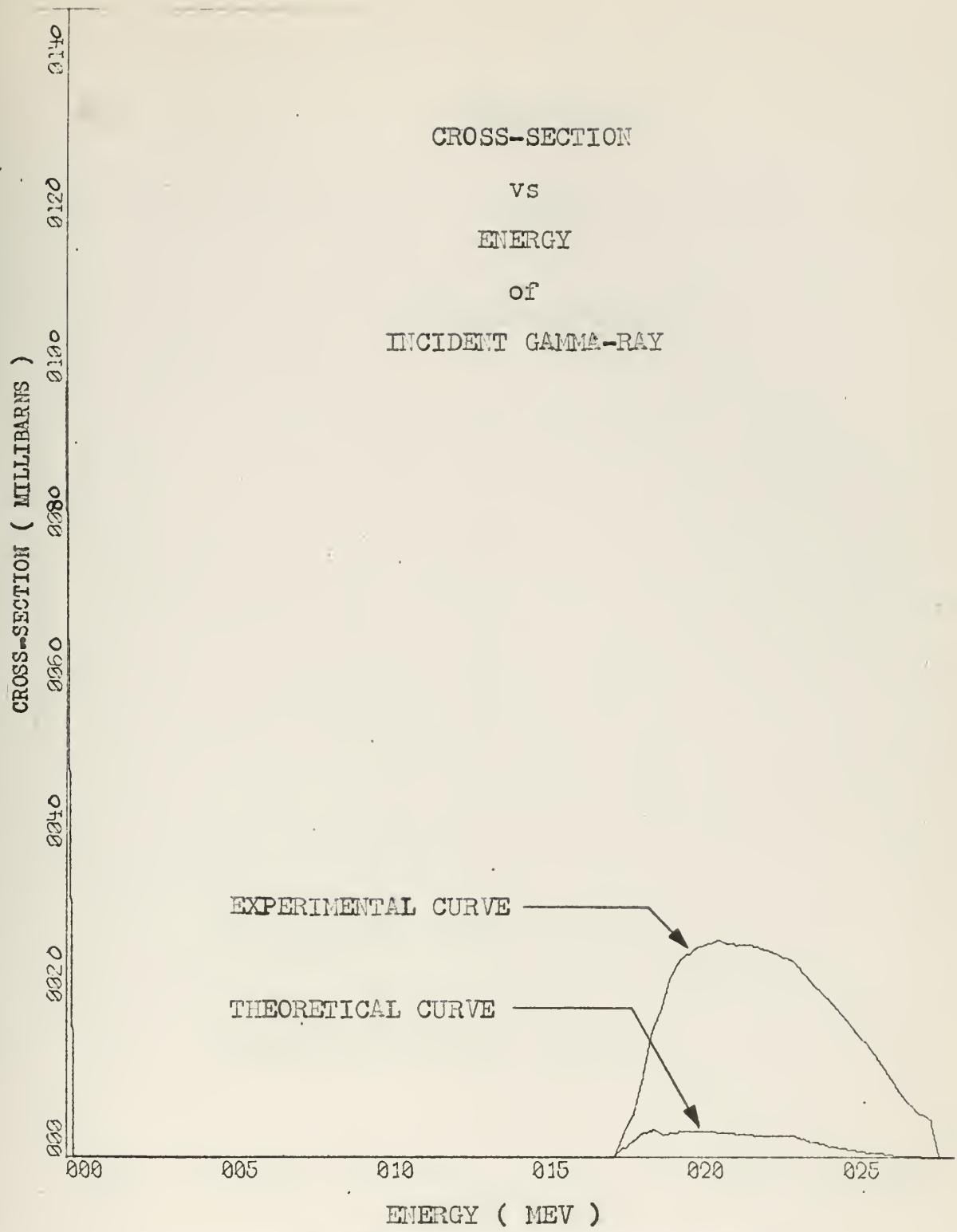
FIGURE 38



DENSITY FUNCTION $P(E) = C E \exp(A E^n)$

$$A = 0.2 \text{ MEV}^{-1}$$

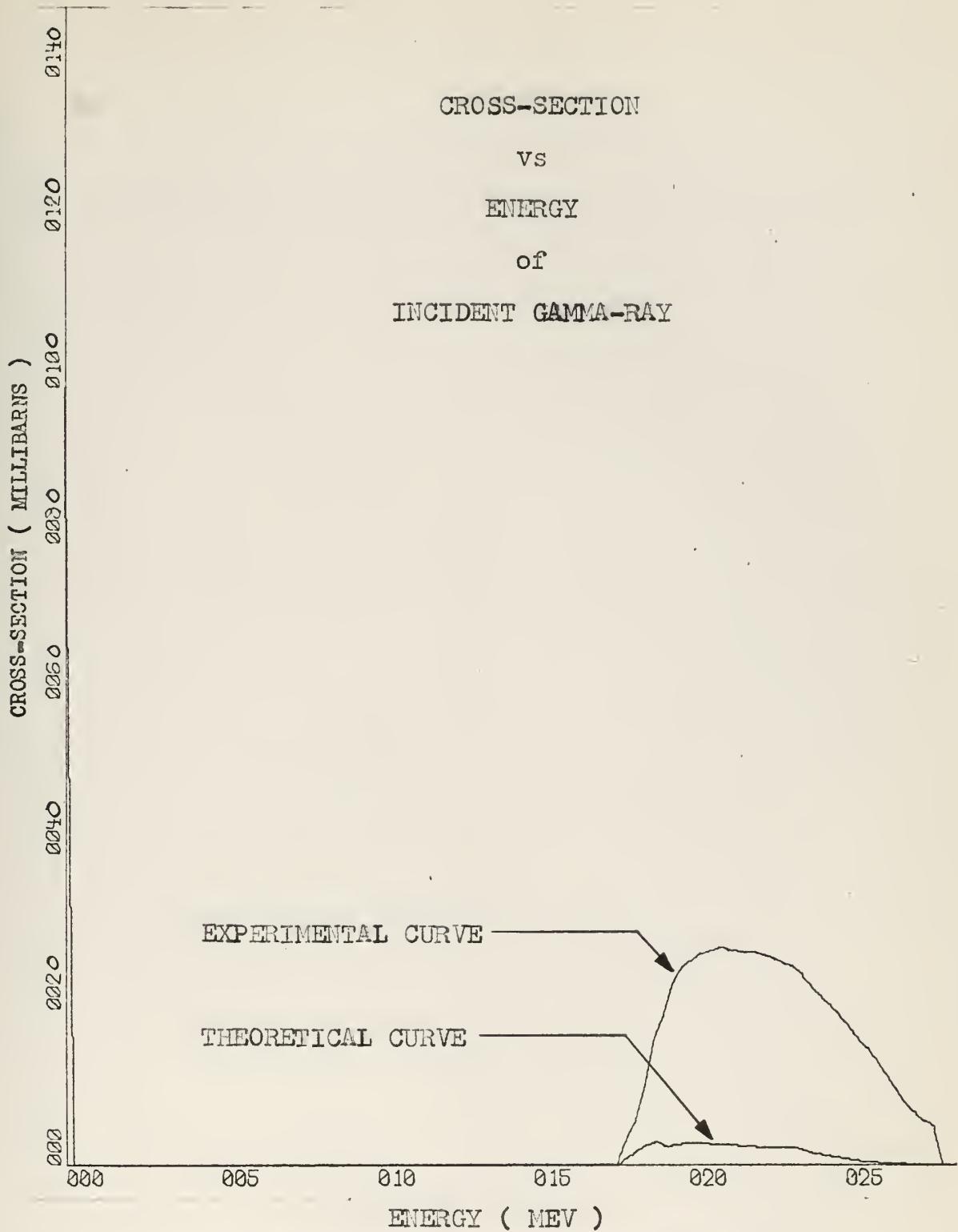
FIGURE 39



$$\text{DENSITY FUNCTION } P(E) = C E \exp(A E^n)$$

$$A = 0.3 \text{ MEV}^{-1}$$

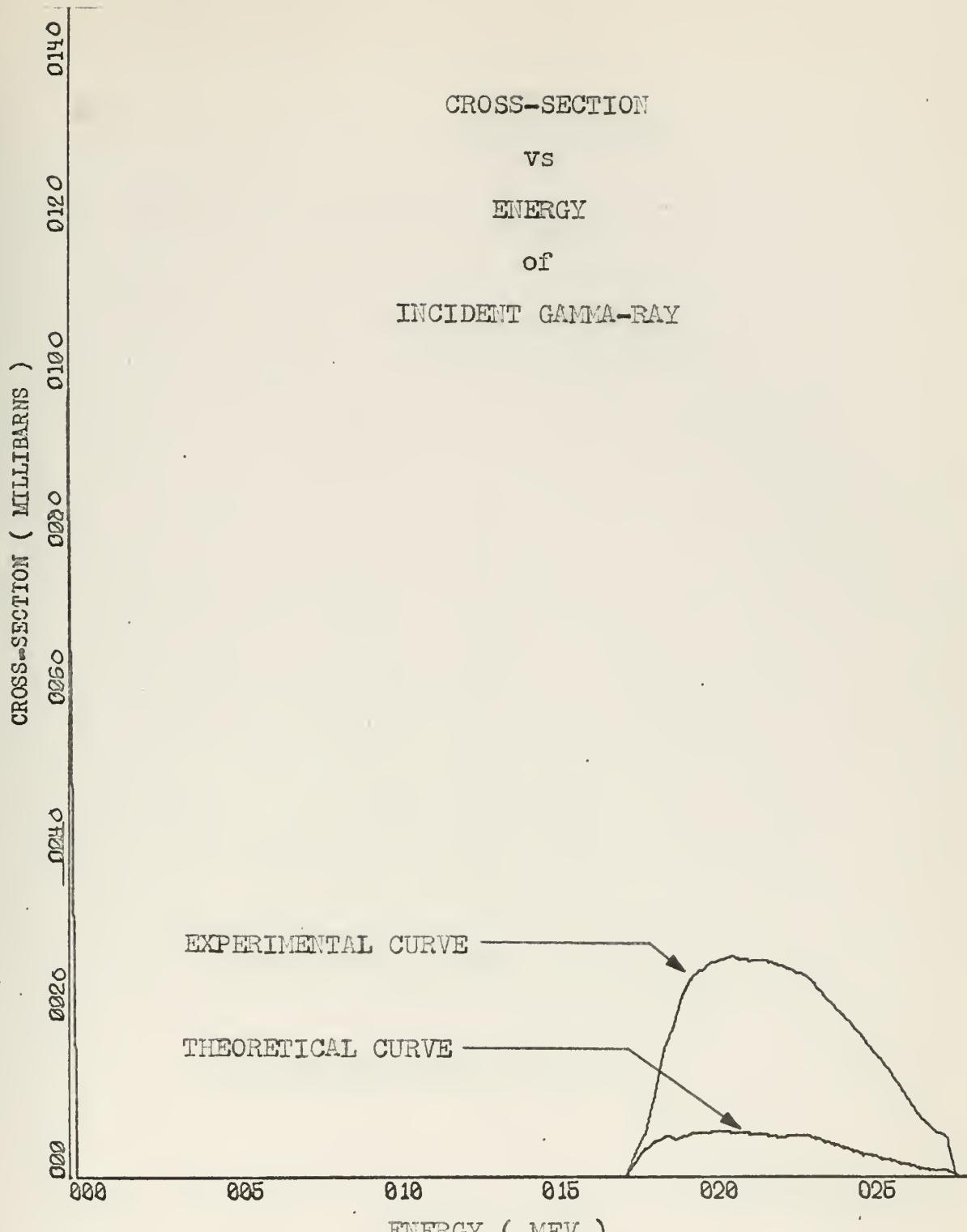
FIGURE 40



$$\text{DENSITY FUNCTION } P(E) = C E \exp(A E^n)$$

$$A = 0.4 \text{ MEV}^{-1}$$

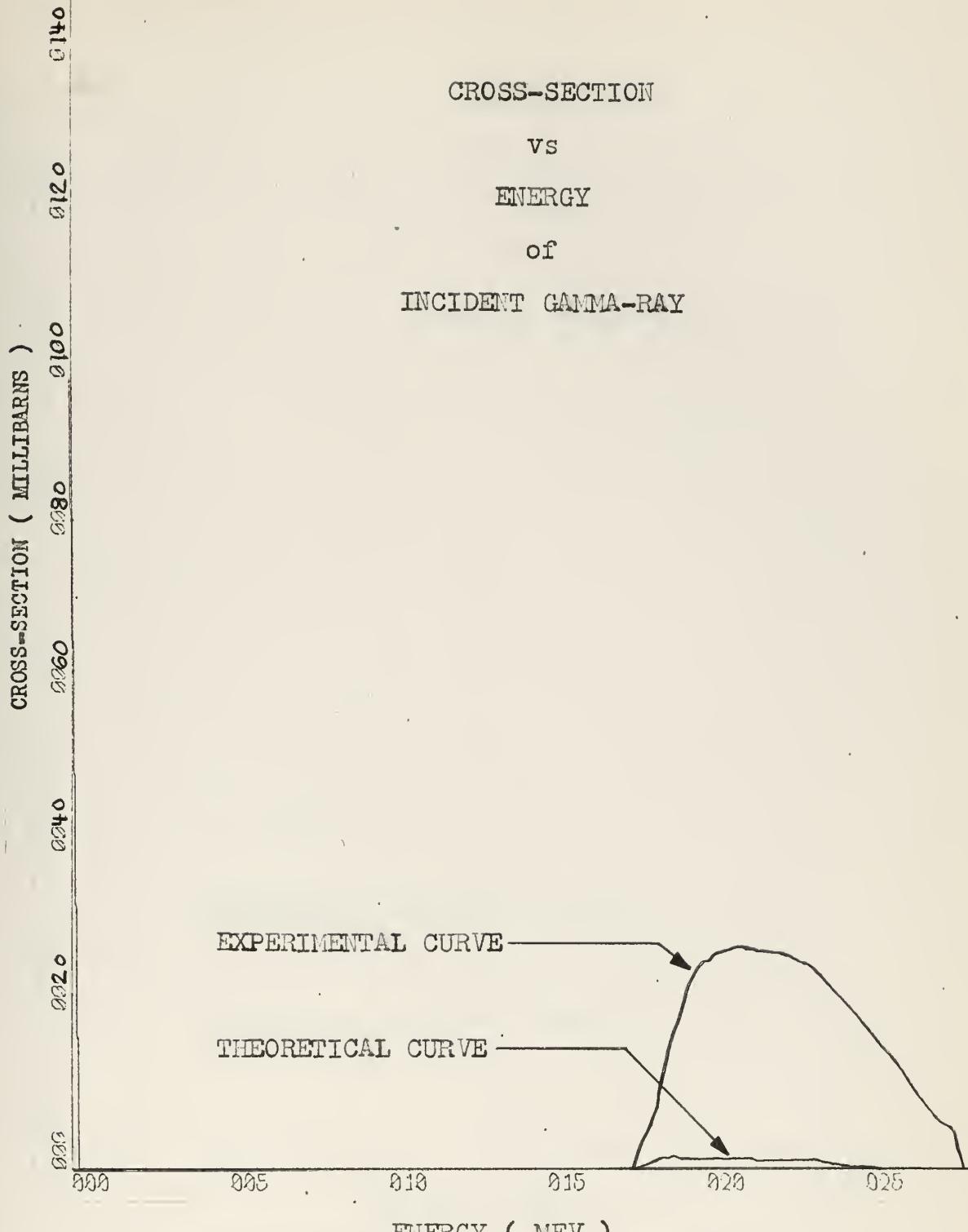
FIGURE 41



$$\text{DENSITY FUNCTION } P(E) = C E \exp(A E^n)$$

$$A = 0.5 \text{ MEV}^{-1}$$

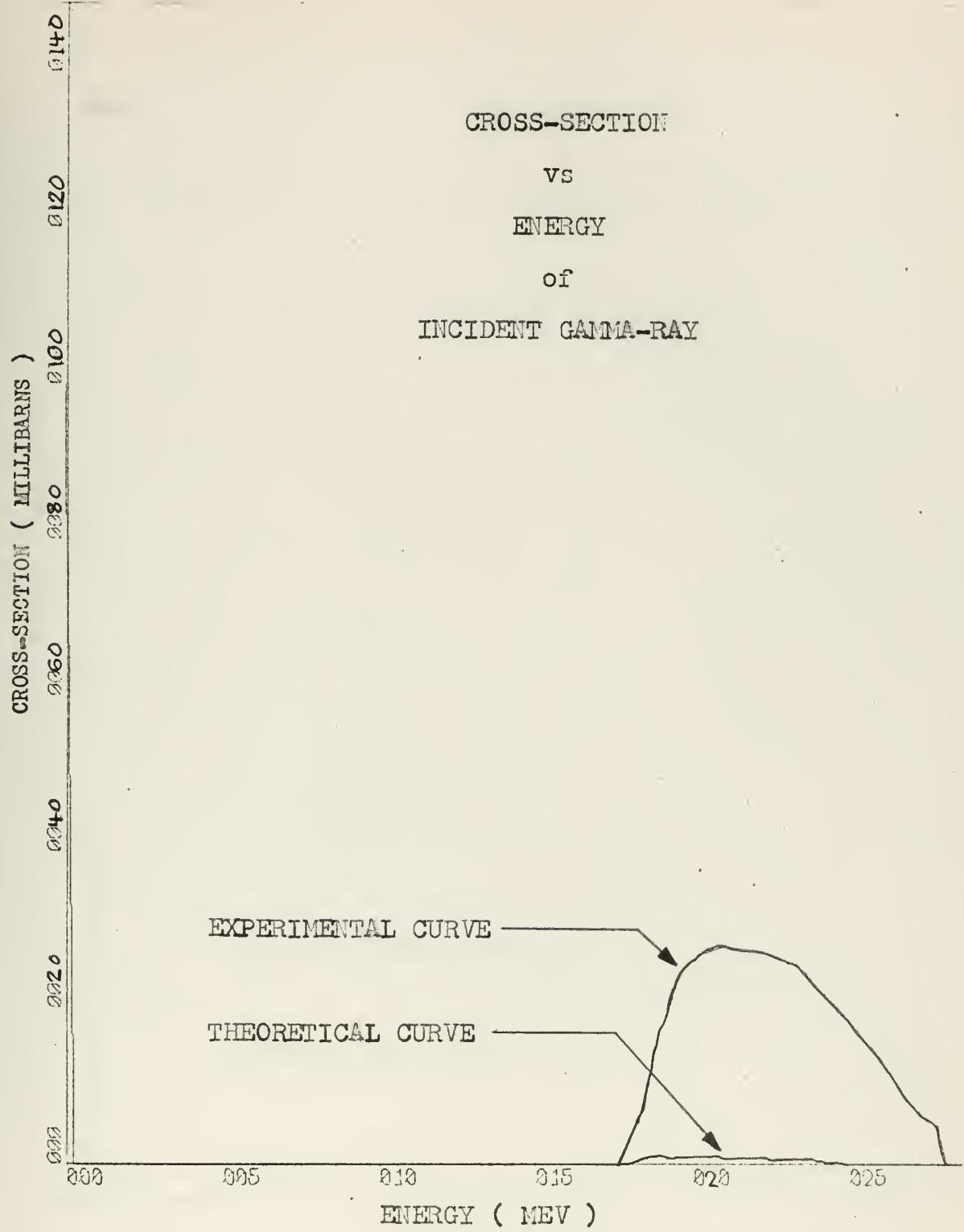
FIGURE 42



DENSITY FUNCTION $P(E) = C E \exp(-A E^n)$

$$A = 0.7 \text{ MEV}^{-1}$$

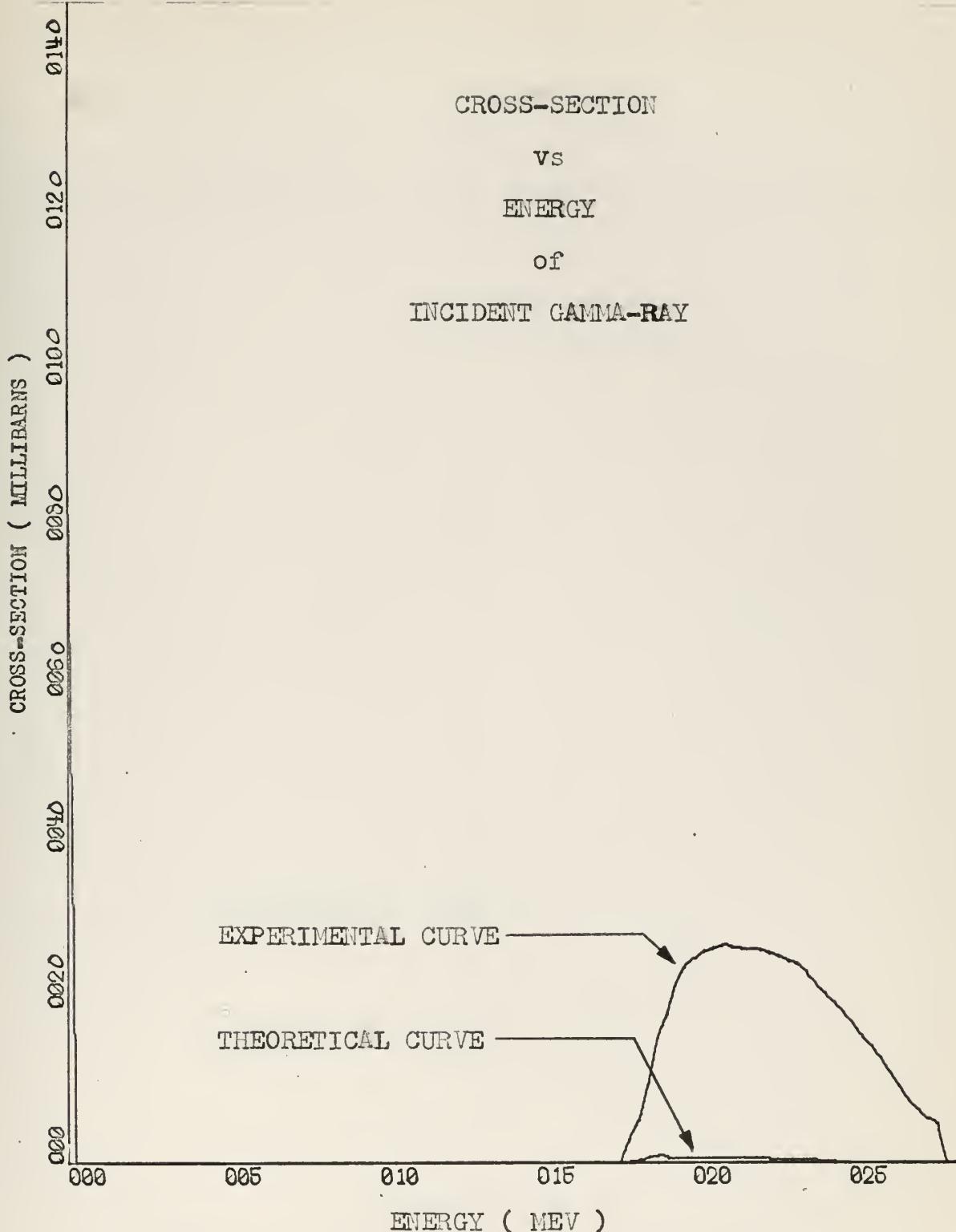
FIGURE 43



DENSITY FUNCTION $P(E) = C E \exp(-A E^n)$

$$A = 0.8 \text{ MEV}^{-1}$$

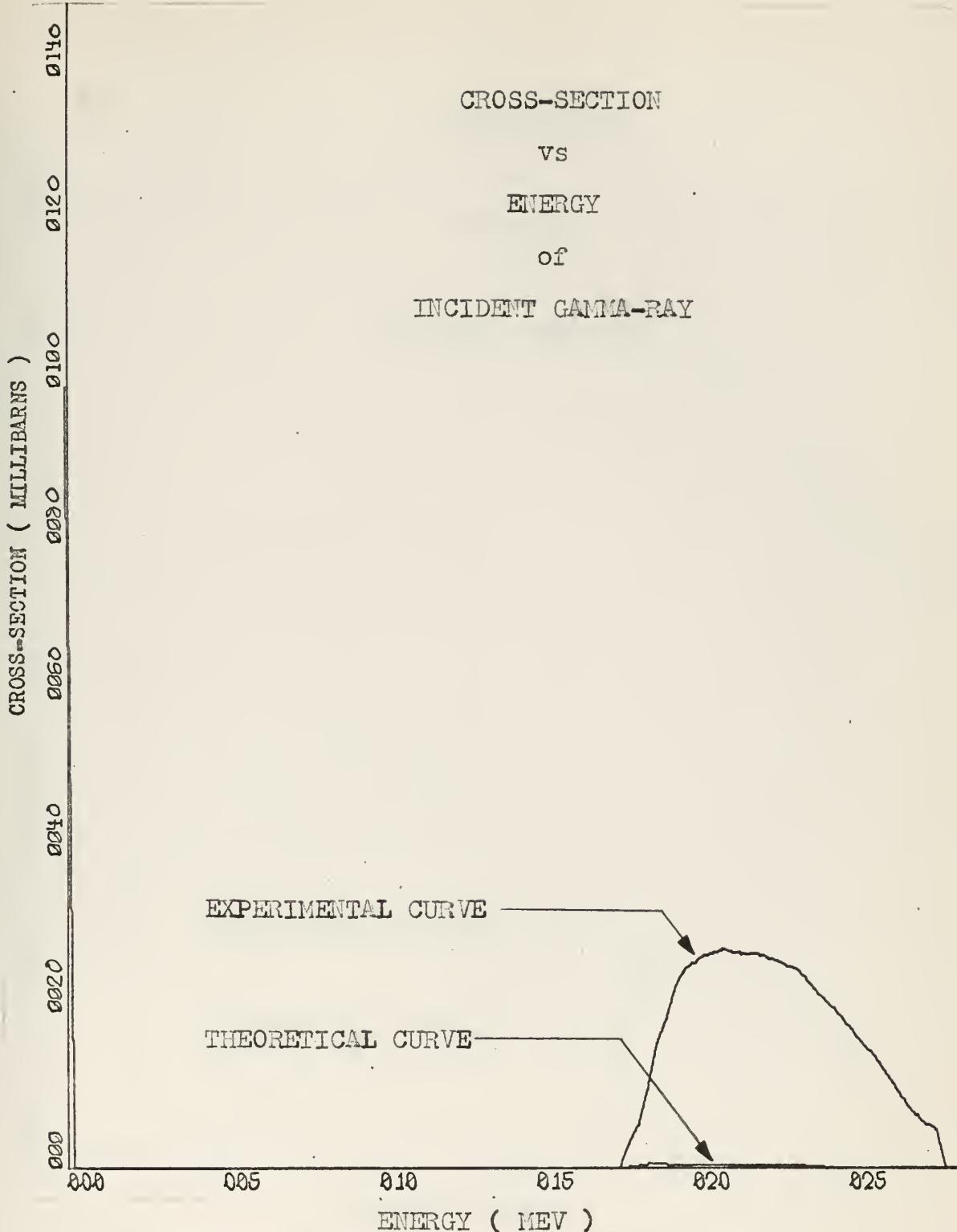
FIGURE 44



$$\text{DENSITY FUNCTION } P(E) = C E \exp(A E^n)$$

$$A = 0.9 \text{ MEV}^{-1}$$

FIGURE 45



$$\text{DENSITY FUNCTION } P(E) = C E \exp(-A E^2)$$

$$A = 1.0 \text{ MEV}^{-1}$$

FIGURE 46

0140
0120
0100
0080
0060
0040
0020
000

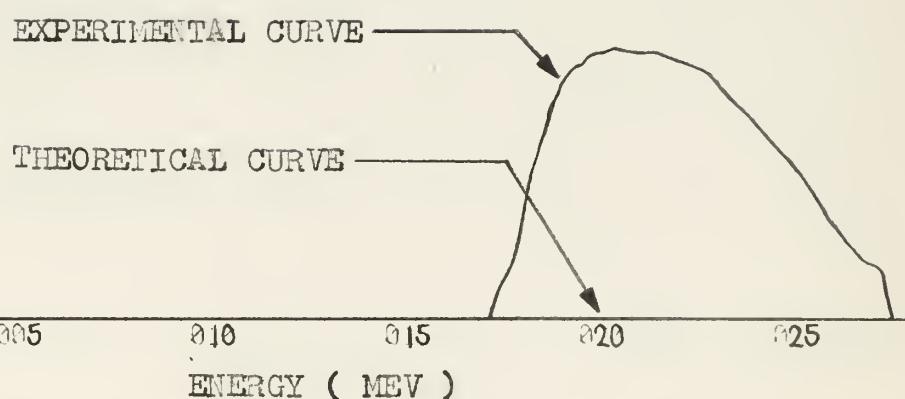
CROSS-SECTION

vs

ENERGY

of

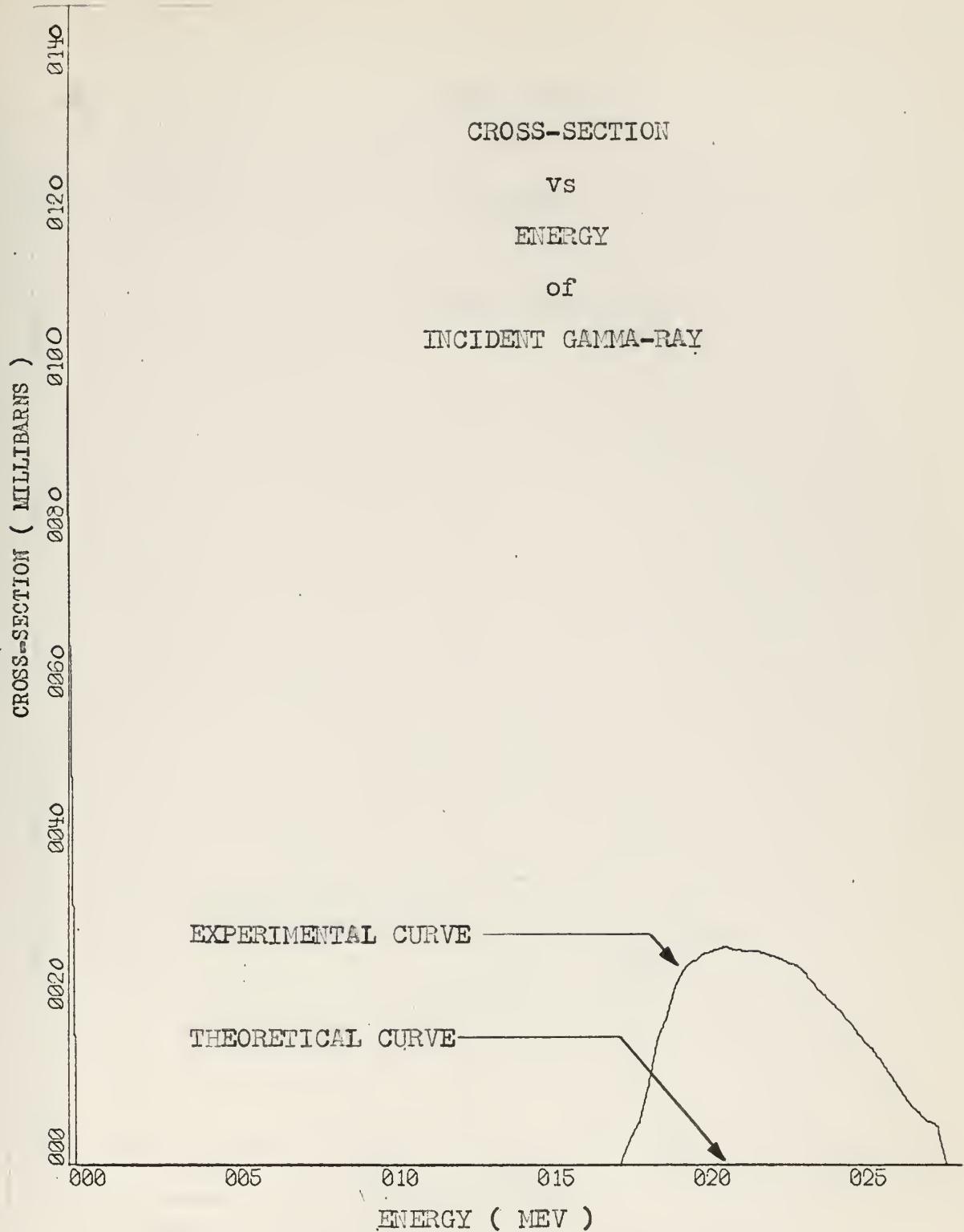
INCIDENT GAMMA-RAY



DENSITY FUNCTION $P(E) = C E \exp(-A E^n)$

$$A = 1.5 \text{ MEV}^{-1}$$

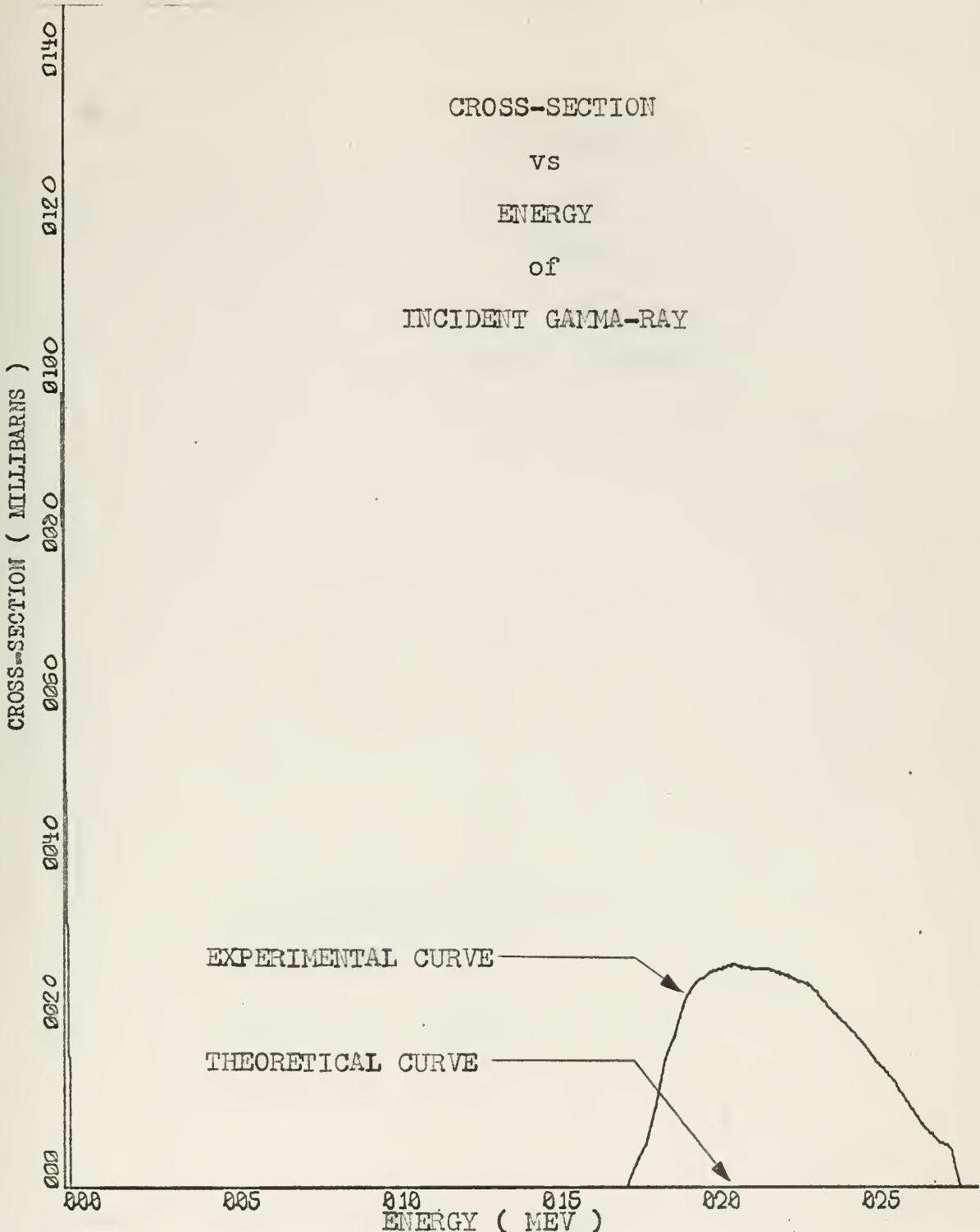
FIGURE 47



$$\text{DENSITY FUNCTION } P(E) = C E \exp(A E^n)$$

$$A = 3.0 \text{ MEV}^{-1}$$

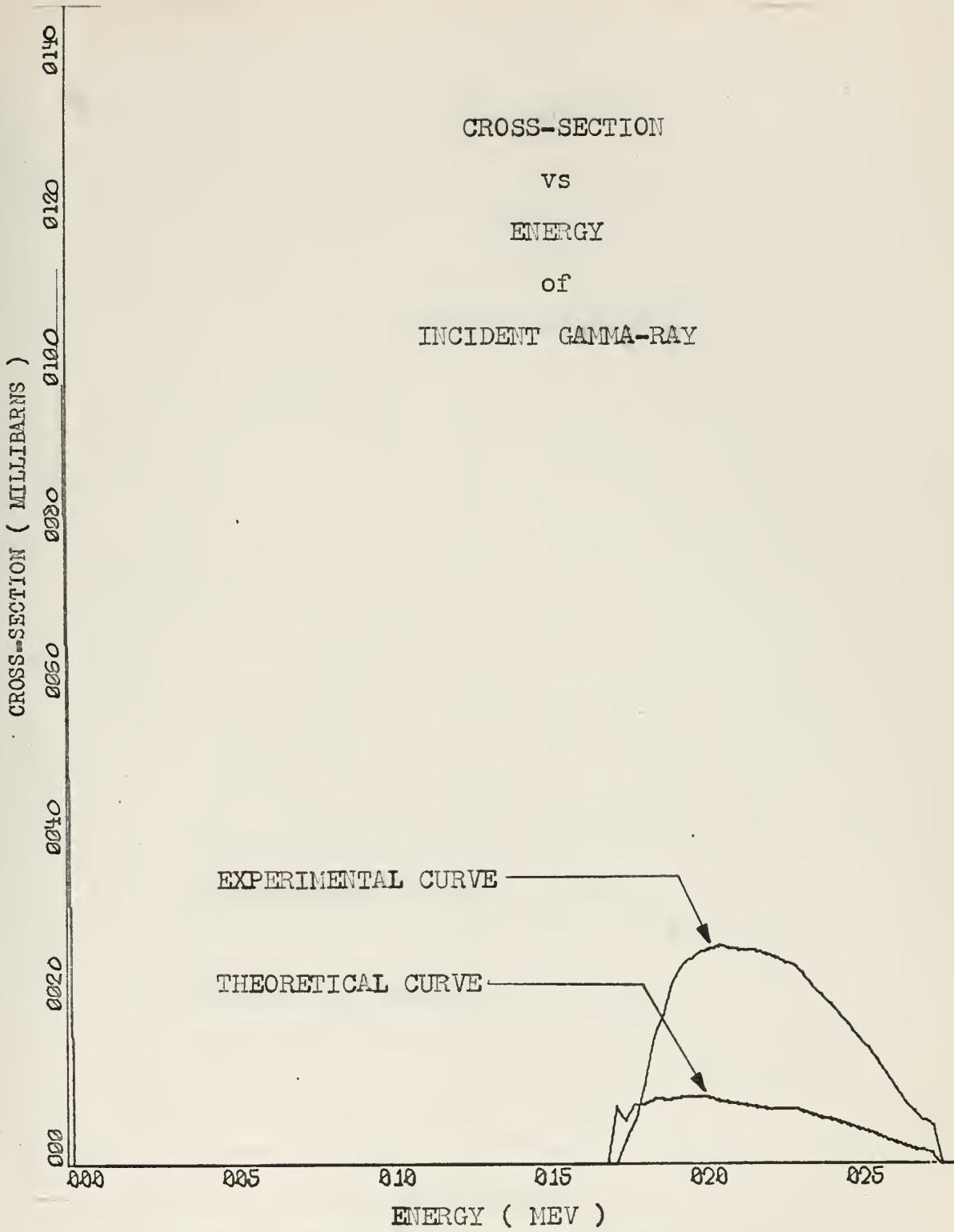
FIGURE 48



$$\text{DENSITY FUNCTION } P(E) = C E \exp(-A E^n)$$

$$A = 10.0 \text{ MEV}^{-1}$$

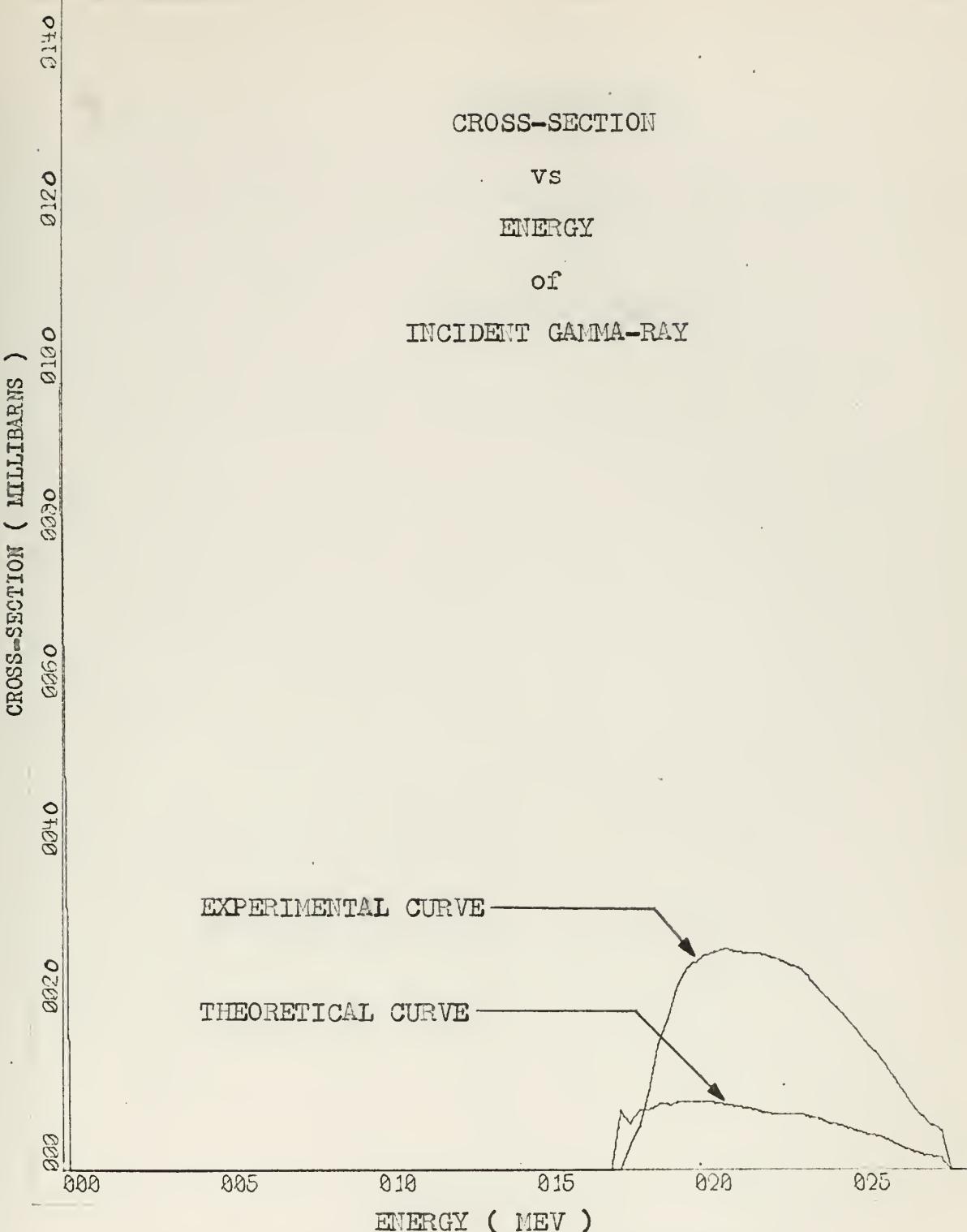
FIGURE 49



$$\text{DENSITY FUNCTION } P(E) = C E / \exp(A E^n)$$

$$A = 0.0001 \text{ MEV}^{-1}$$

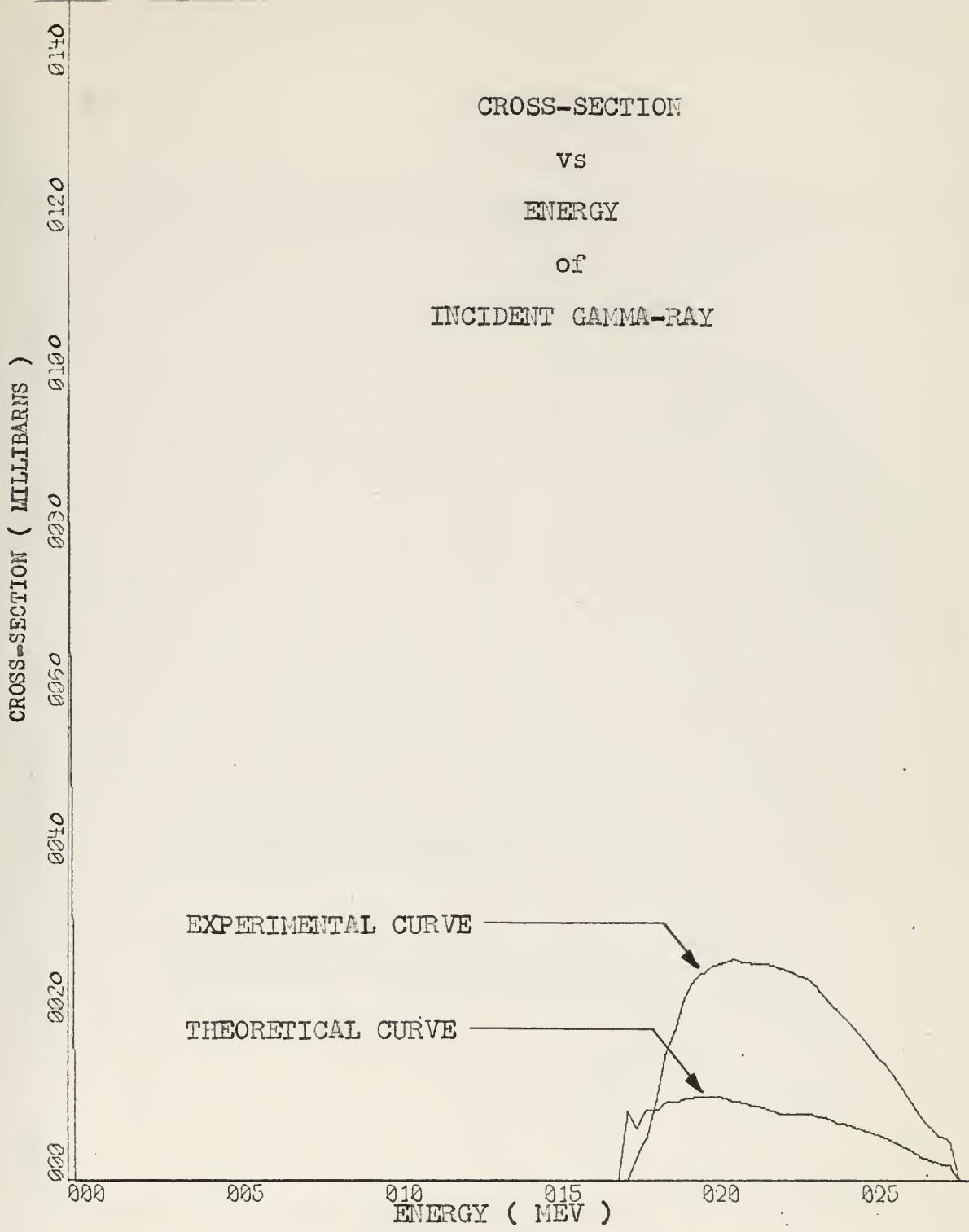
FIGURE 50



DENSITY FUNCTION $P(E) = C E / \exp(A E^n)$

$$A = 0.01 \text{ MEV}^{-1}$$

FIGURE 51

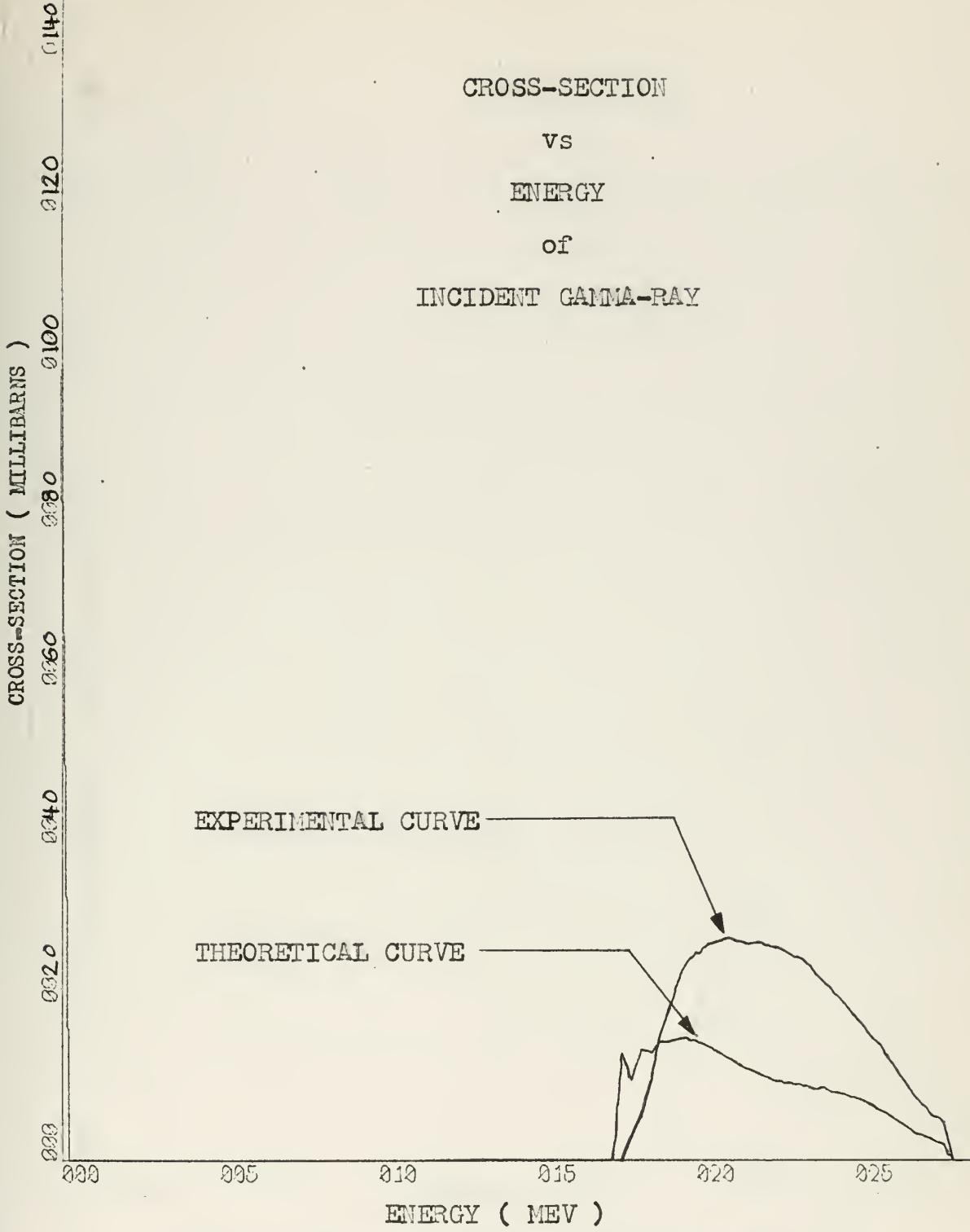


$$\text{DENSITY FUNCTION } P(E) \equiv C E / \exp(A E^n)$$

$$A = 0.2 \text{ MEV}^{-1}$$

FIGURE 52

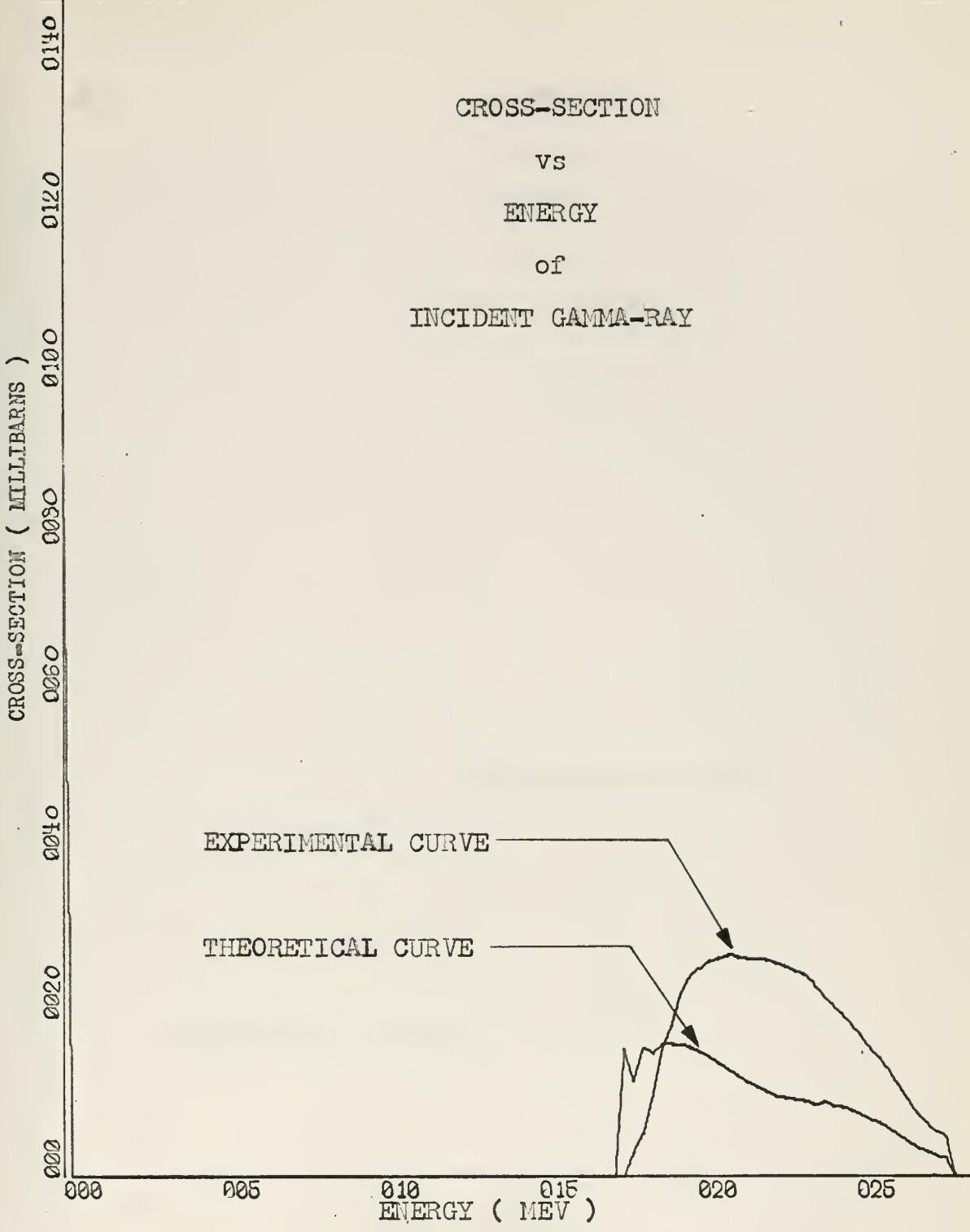
CROSS-SECTION
vs
ENERGY
of
INCIDENT GAMMA-RAY



$$\text{DENSITY FUNCTION } P(E) = C E / \exp(A E^n)$$

$$A = 1.0 \text{ MEV}^{-1}$$

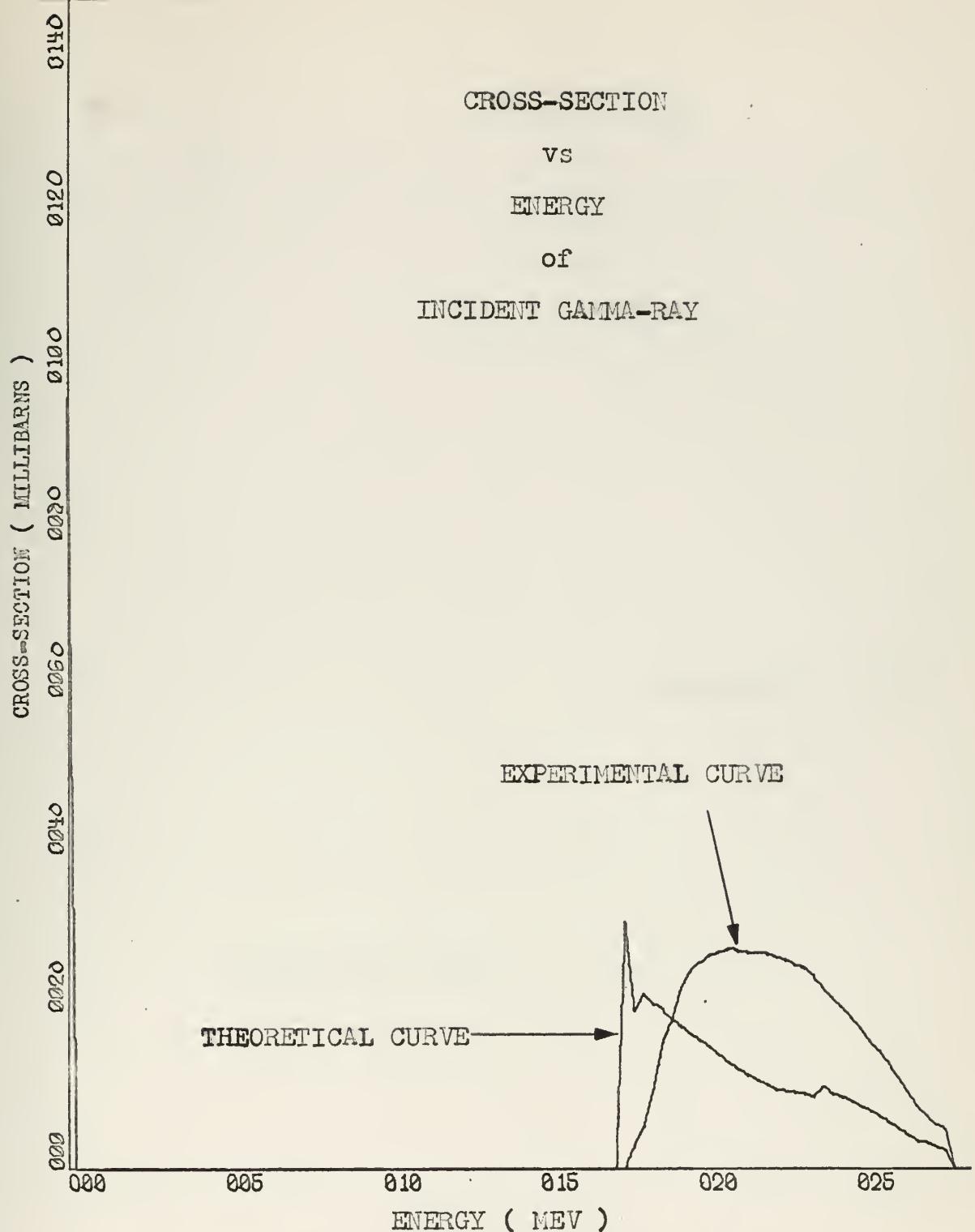
FIGURE 53



$$\text{DENSITY FUNCTION } P(E) = C E / \exp(A E^n)$$

$$A = 1.5 \text{ MEV}^{-1}$$

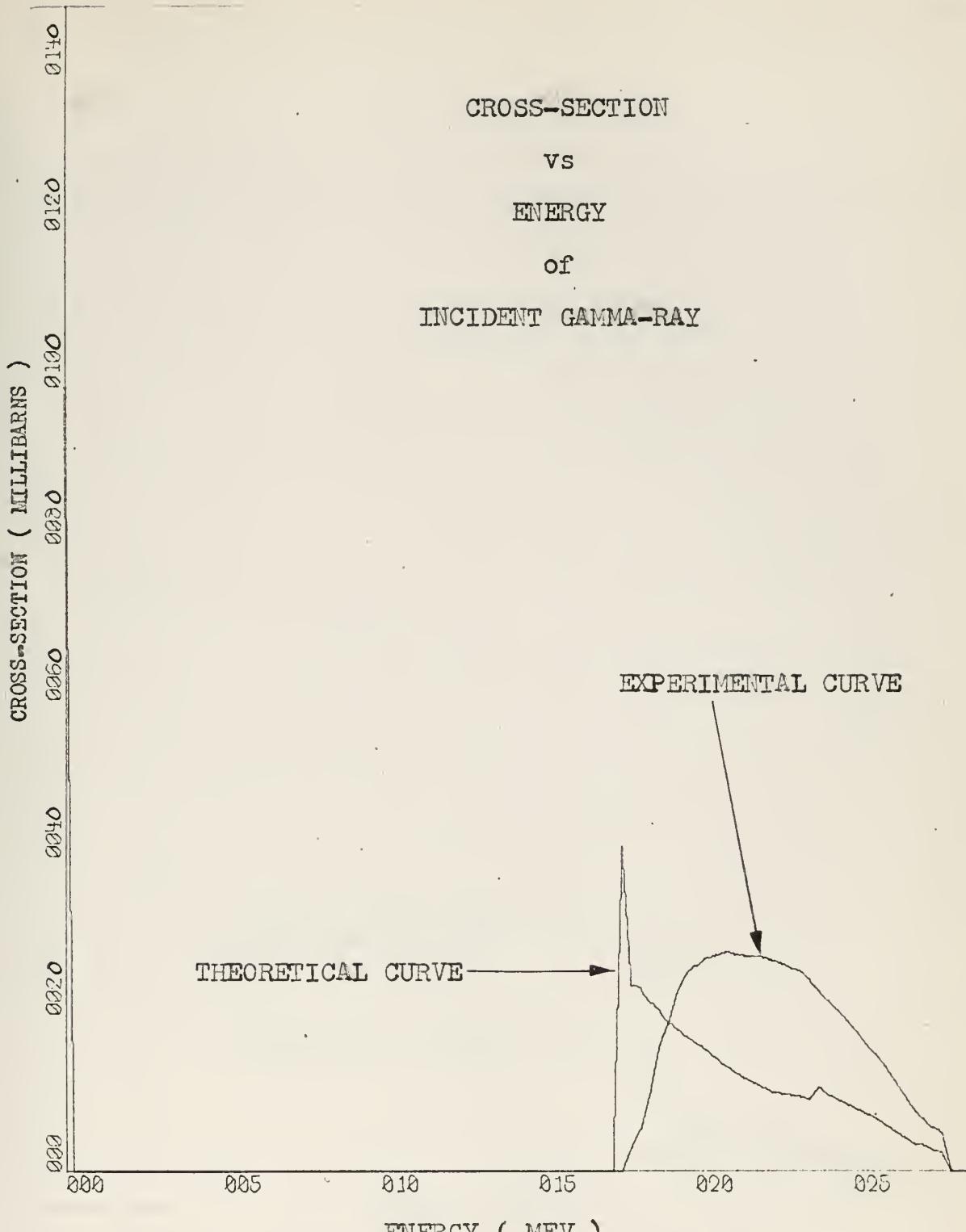
FIGURE 54



$$\text{DENSITY FUNCTION } P(E) = C E / \exp(A E^n)$$

$$A = 5.0 \text{ MEV}^{-1}$$

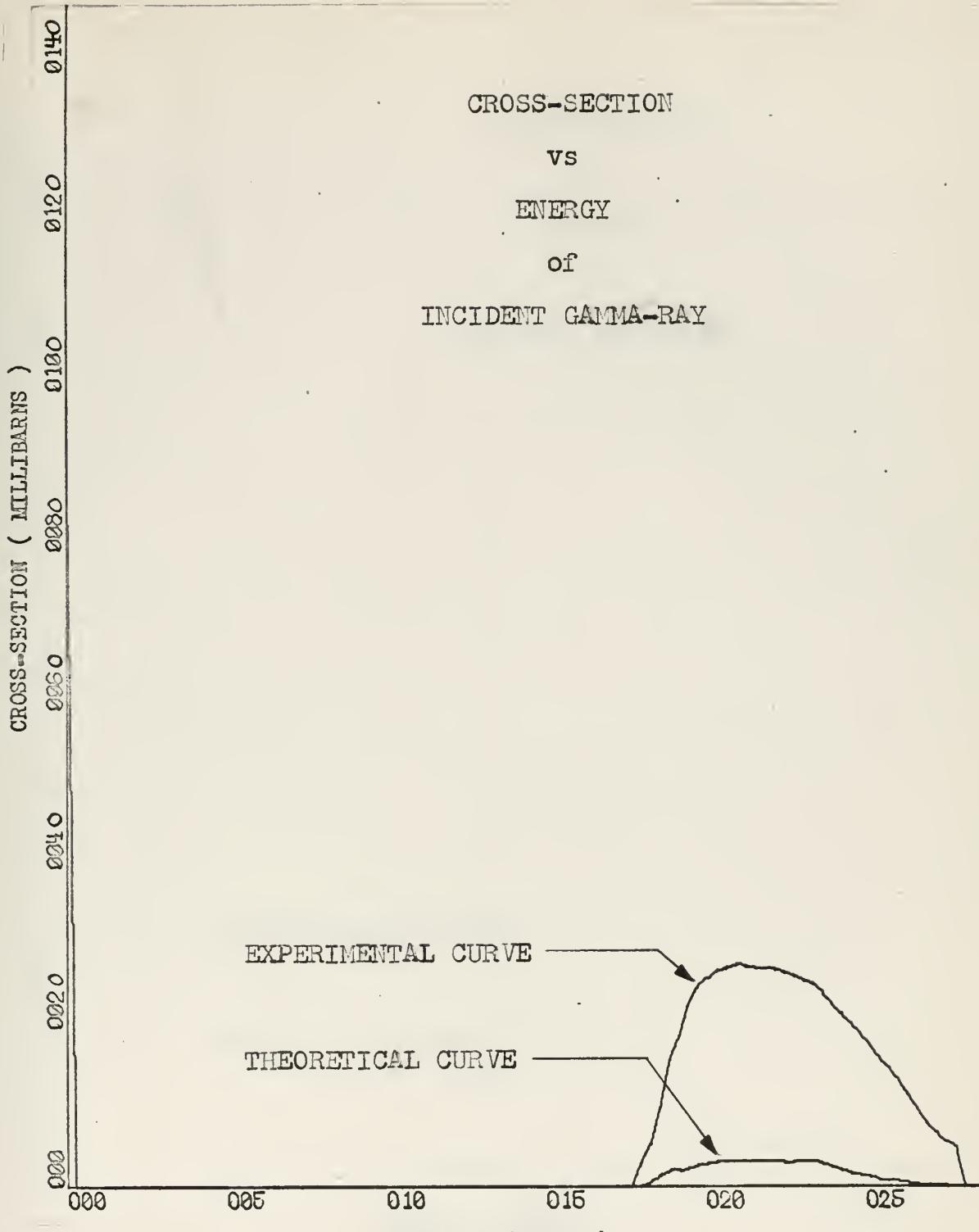
FIGURE 55



$$\text{DENSITY FUNCTION } P(E) = C E / \exp(A E^n)$$

$$A = 10.0 \text{ MEV}^{-1}$$

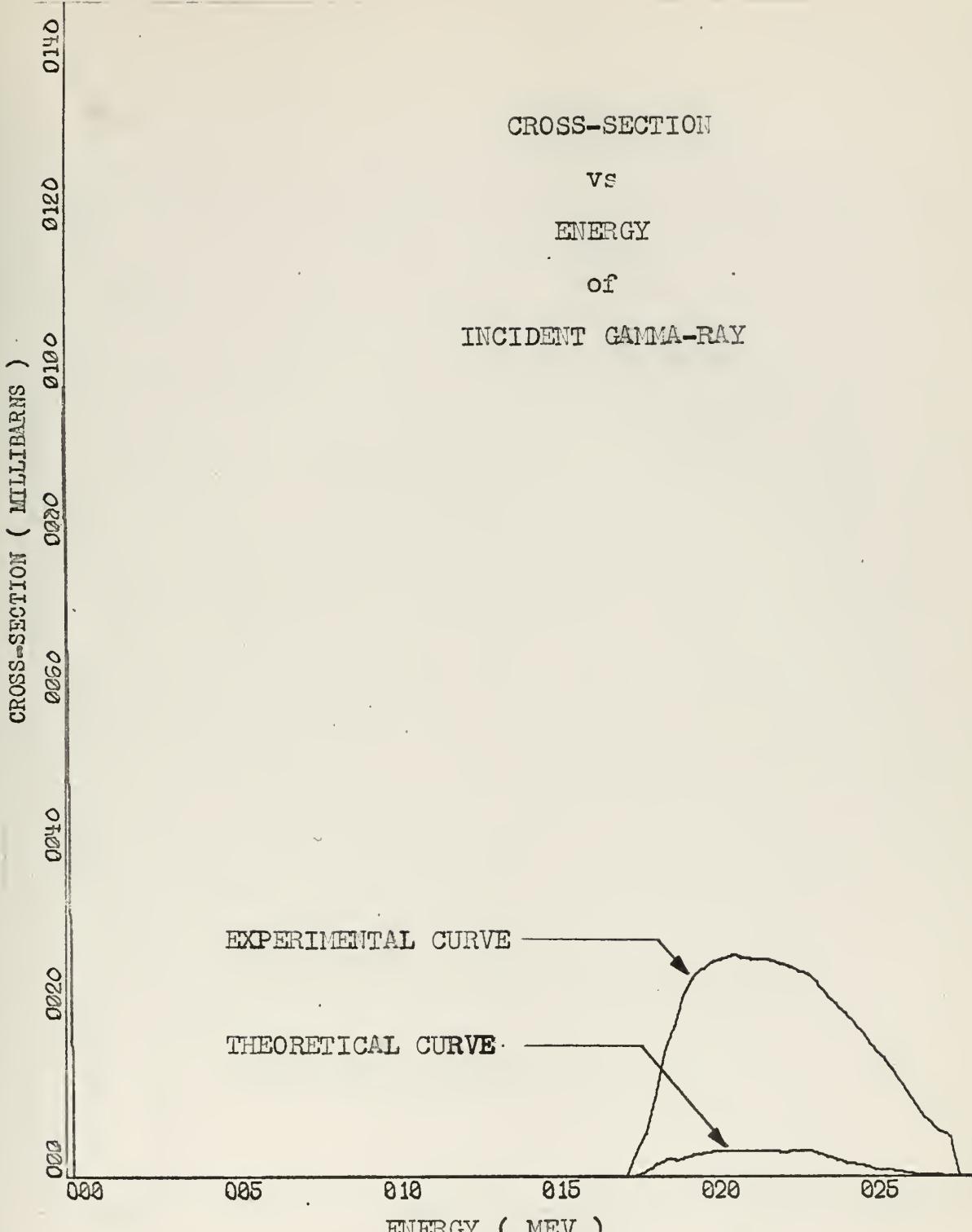
FIGURE 56



$$\text{DENSITY FUNCTION } P(E) = C(E)^3 \exp(A E^n)$$

$$A = 0.0001 \text{ MEV}^{-1}$$

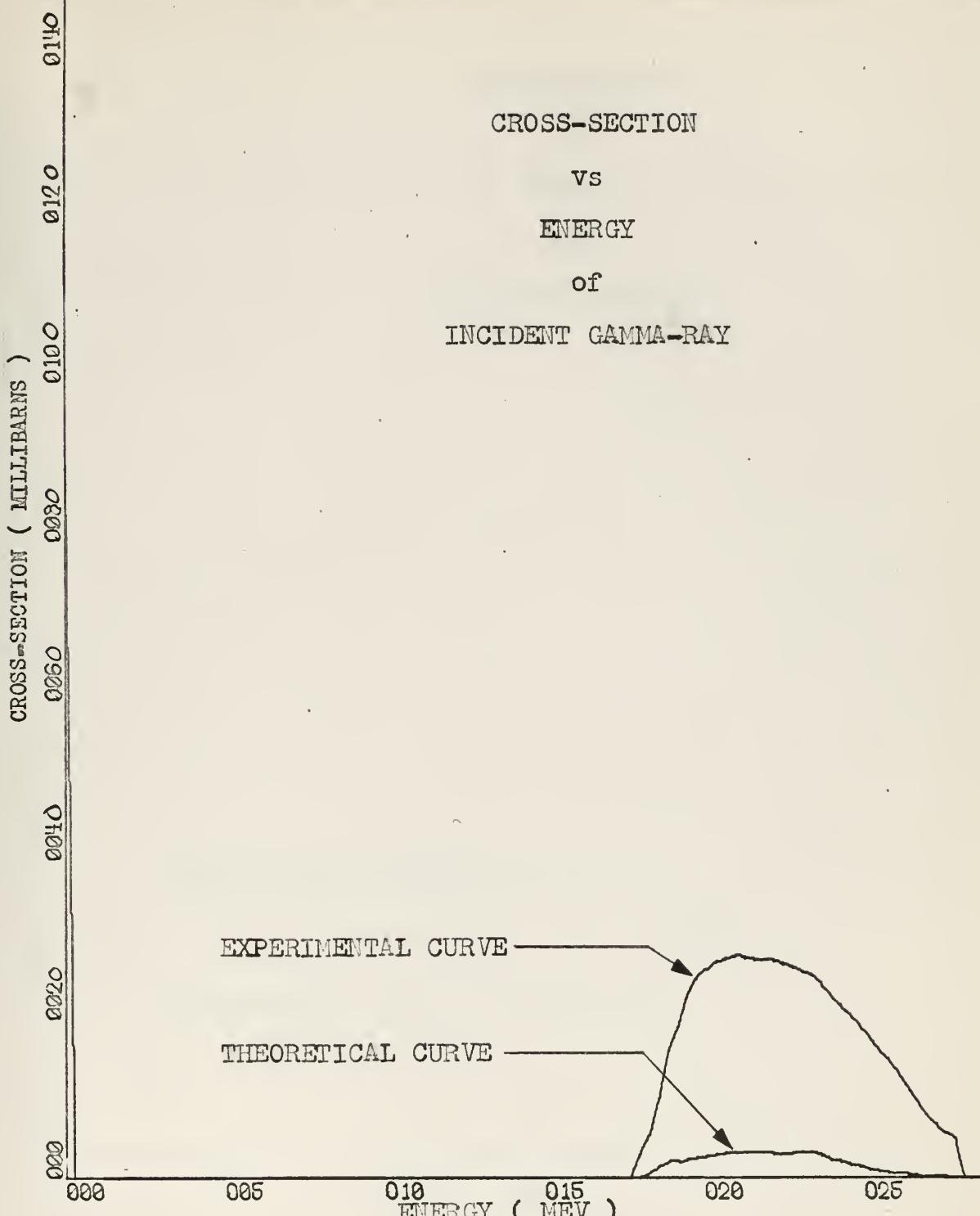
FIGURE 57



$$\text{DENSITY FUNCTION } P(E) = C(E)^3 \exp(-A E^n)$$

$$A = 0.001 \text{ MEV}^{-1}$$

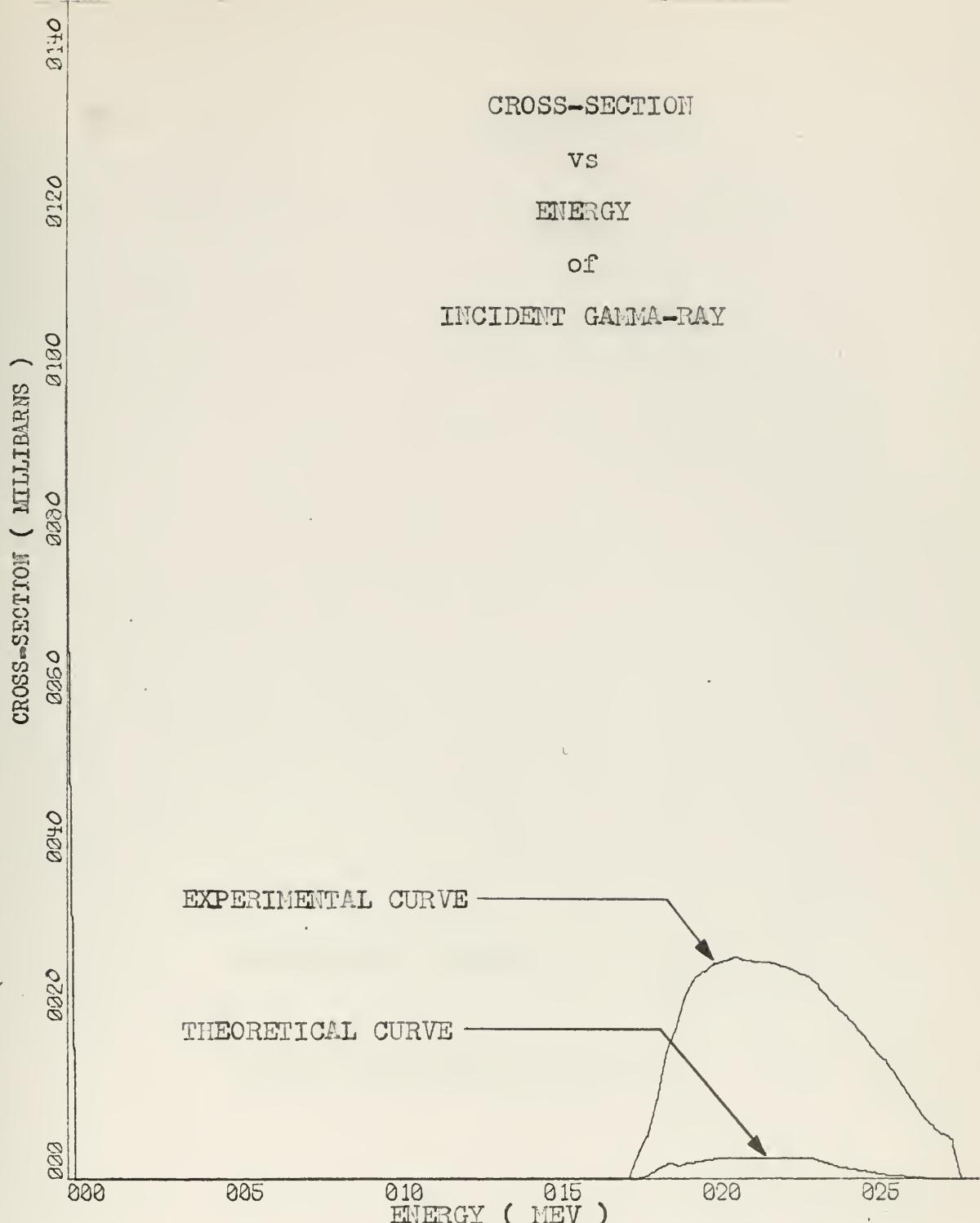
FIGURE 58



DENSITY FUNCTION $P(E) \equiv C(E)^3 \exp(-A E^n)$

$$A \equiv 0.01 \text{ MEV}^{-1}$$

FIGURE 59



DENSITY FUNCTION $P(E) = C(E)^3 \exp(-AE^n)$

$$A = 0.1 \text{ MEV}^{-1}$$

FIGURE 60

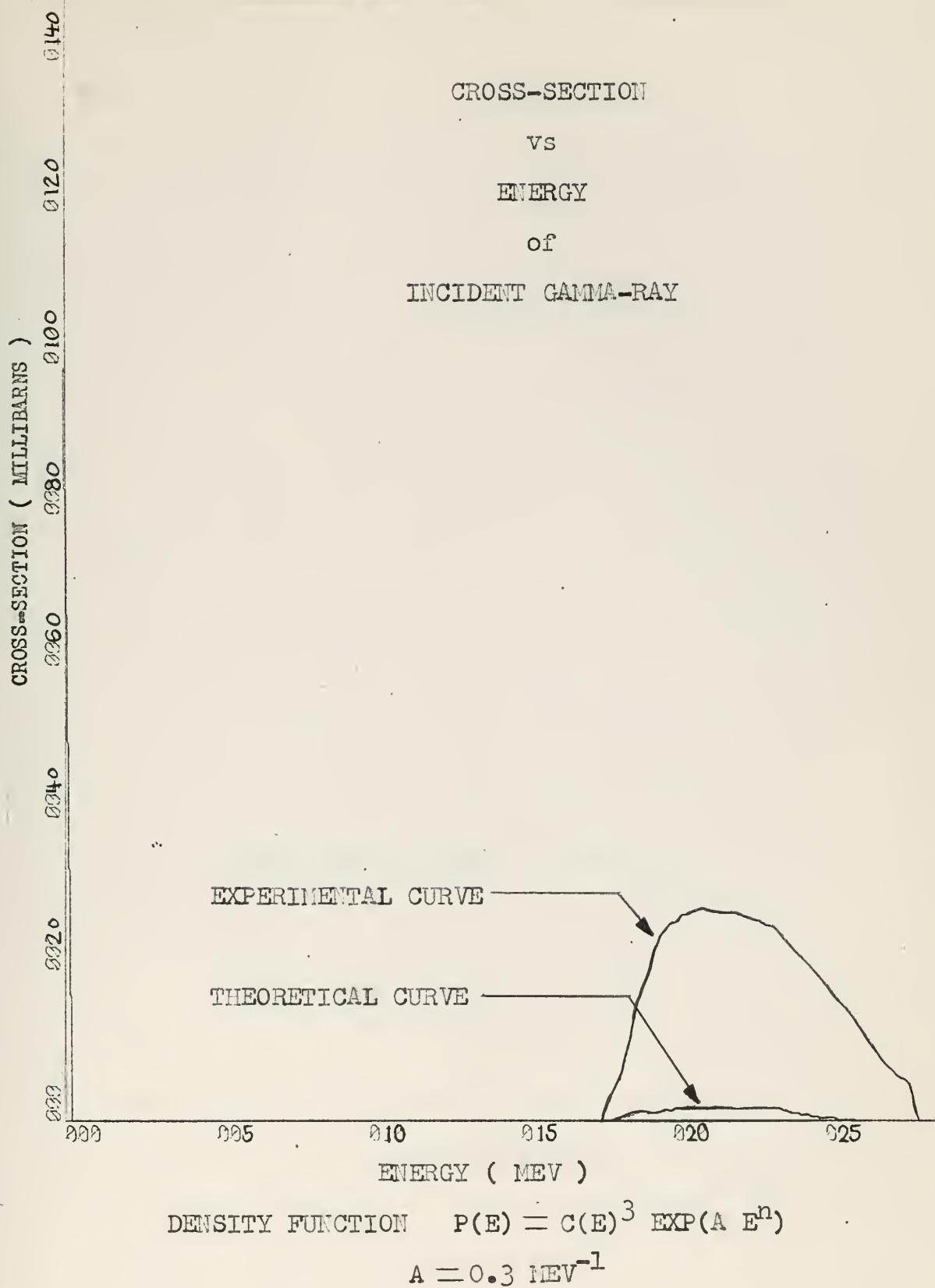
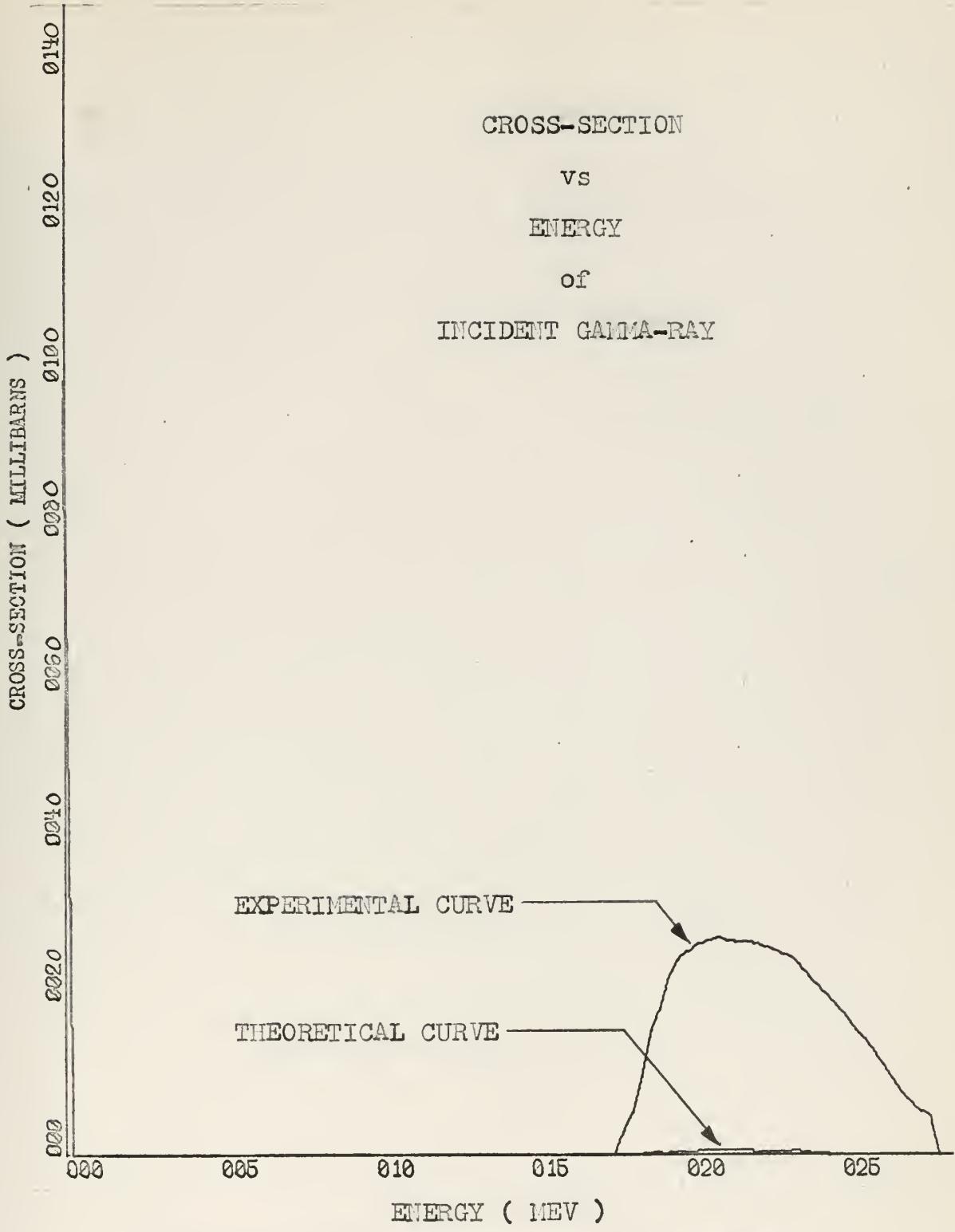


FIGURE 61

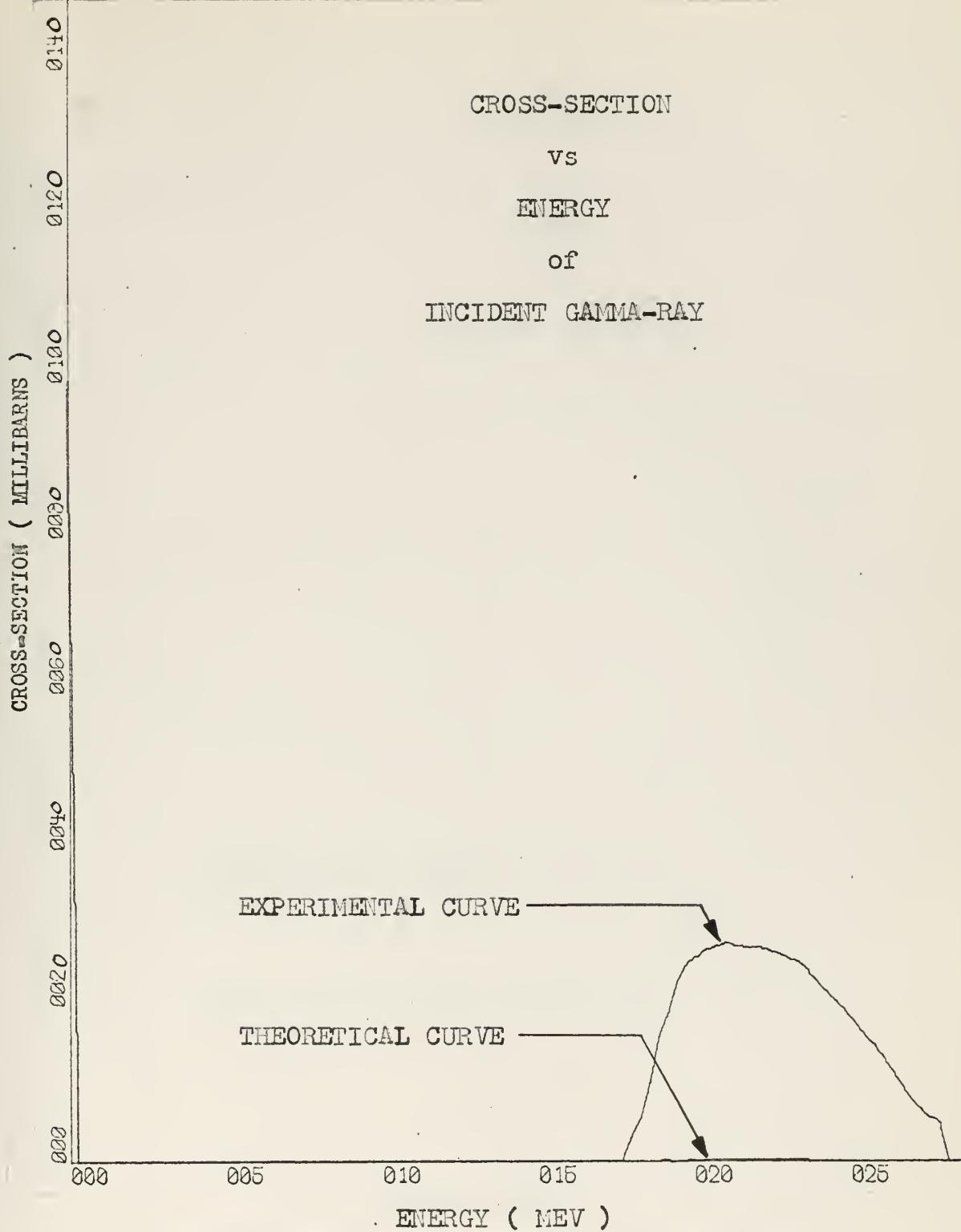
CROSS-SECTION
vs
ENERGY
of
INCIDENT GAMMA-RAY



DENSITY FUNCTION $P(E) = C(E)^3 \exp(A E^n)$

$$A = 0.7 \text{ MEV}^{-1}$$

FIGURE 62



$$\text{DENSITY FUNCTION } P(E) = C(E)^3 \exp(-A E^n)$$

$$A = 1.0 \text{ MEV}^{-1}$$

FIGURE 63

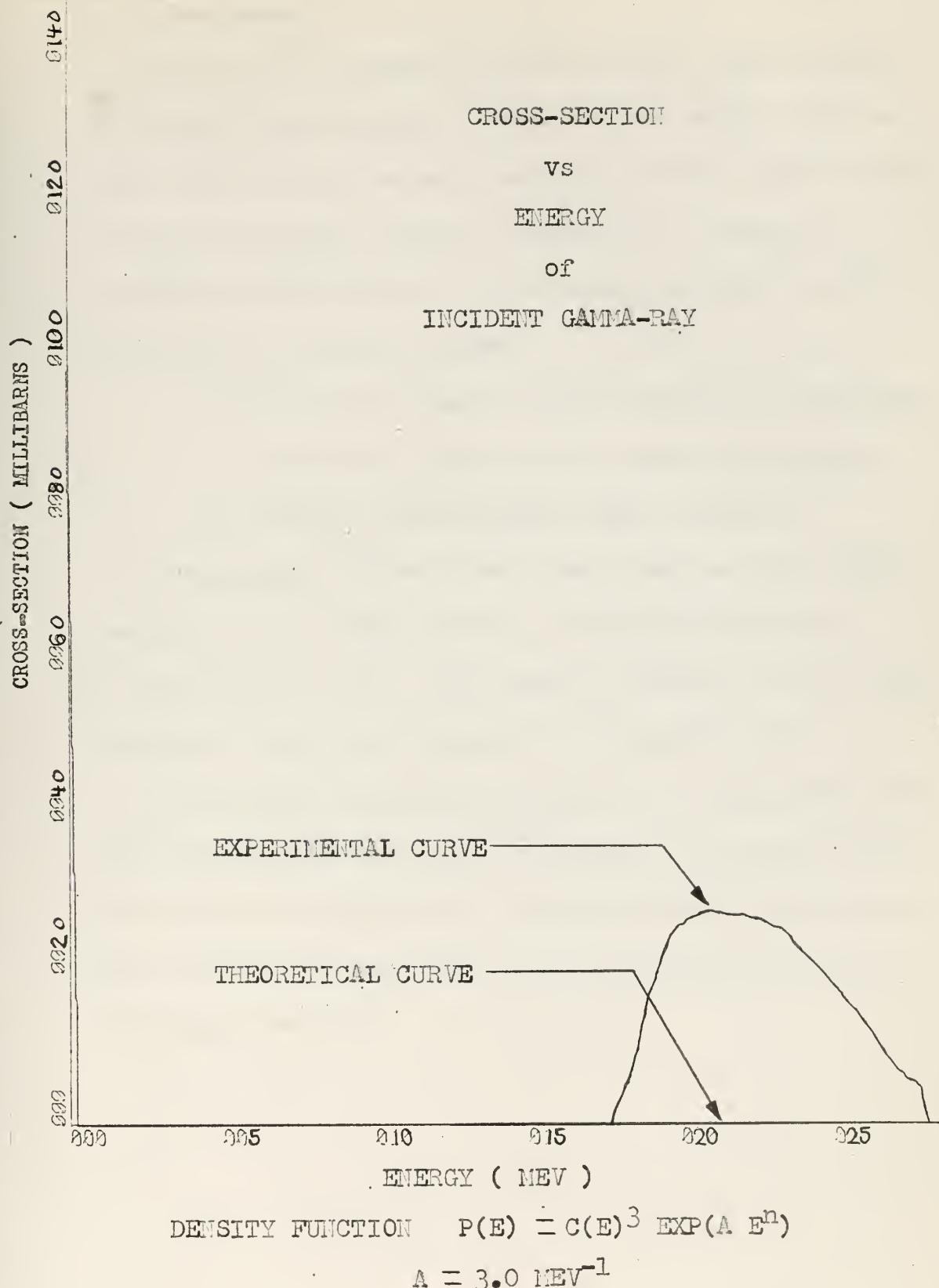


FIGURE 64

6. Conclusions

None of the density functions tested thus far give an adequate approximation to the experimentally obtained data. Two of the tested functions produce a good shape, but in all cases the maximum values do not approach the experimental values. The lack of conformity so far may be due to several causes:

- a. The proper density function has not been found.
- b. Coulomb and other effects have been ignored.
- c. Tertiary reactions have been omitted.

The results indicate that there may be some power function of (E) which acts as a variable coefficient of the exponential term. The density function with the $(E)^3$ coefficient is a first attempt to find such a term.

The results obtained, although not in close agreement with experimental data, are encouraging and indicate that this model with appropriate modifications and development can be used for making useful statistical approximations for nuclear reactions.

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APPENDIX I

CROSS-SECTION PROGRAM

This program was written for the purpose of calculating cross-sections in the decay of the compound nucleus. Although written for the $X(\gamma, 2n)Y$ reaction with Cu⁶⁵ as the target nucleus, this program can be used for any nucleus in computing an approximation to the cross-sections for the emission of particles. Appropriate assumptions must be made with regard to the coulomb barrier, spin, angular momentum, etc.

In general, the program consists of a series of calculations in which three subroutines perform most of the mathematical operations. These subroutines calculate values of single and double integrals. A subroutine for the computation of triple integrals is included, but was not used in the main program.

Subroutine PROD calculates the values of single integrals. The call statement is:

```
CALL PROD( EL, PIN, PNO, NWB)
```

The variables in the call statement have the following meaning:

EL is the threshold for the reaction being calculated.

PIN is a constant used in calculating double and

triple integrals. For the single integral, PIN equals 1.0.

NWB is the index corresponding to the energy range over which the calculation is made. It is determined by dividing the energy in MEV by 0.3. (e.g., NWB = E/.3).

It is appropriate to note here that all threshold energies have been adjusted so as to be divisible by 0.3 in order to simplify the computer operations.

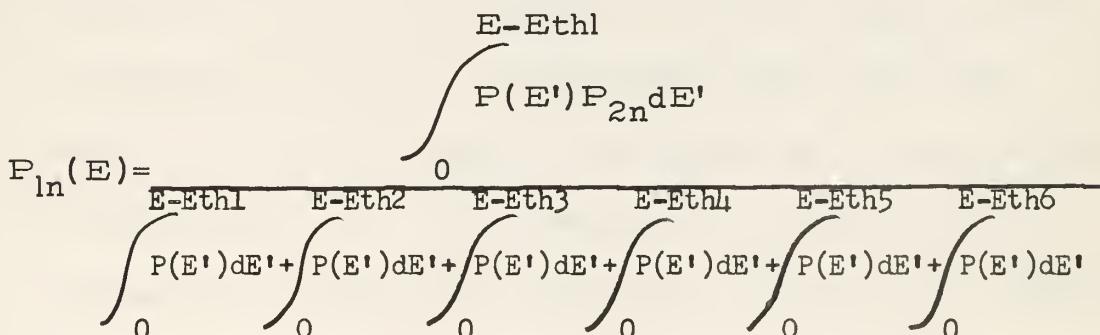
Subroutine GRASP calculates the values of double integrals. The call statement is:

```
CALL GRASP (EL1,EL2,EL3,EL4,EL5,EL6,  
PN20,POCN,PNLN,EPU,NPT,NPI,NP2,NP3,NP4,NP5,  
NP6).
```

In order to observe more clearly the meaning of each variable, consider a two step reaction:



For the total reaction of $X + \gamma \rightarrow Z + 2n$ over the energy range of 0 to 30 MEV, six reactions are possible. Let the probability for the first reaction be:



Now $P_{NLN} = PNL1$ corresponding to the Eth1 for the threshold one reaction in the numerator. EL1 through EL6 are the threshold energies for each reaction that can take place over the prescribed energy range. EPL is the threshold energy for the initial reaction; in this example EHL = EL1.

Next, consider the second reaction:

$$P_{2n}(E) = \frac{\int_0^{E - Eth3} R(E - Eth5) P(E3') dE3'}{\int_0^{E - Eth3} P(E3') dE3' + \int_0^{E - Eth4} P(E3') dE3' + \int_0^{E - Eth6} P(E3') dE3'}$$

In the call statement, POCN = POC3 corresponding to the Eth3 for the threshold three reaction in the numerator. EPU is the threshold for the second reaction and in this example EPU= EL3. The variables NPI through NP6 are indices for the computation of the integrals in the denominator. If a particular integral is not to appear in the denominator, then the corresponding index will be set equal to 2; if the integral is to appear in the denominator, the index will be 1. For example, in the probability for the second reaction the integral for Eth1 does not appear in the denominator; therefore, NPI will equal 2 in the call statement. For the above probability, the sequence NPI

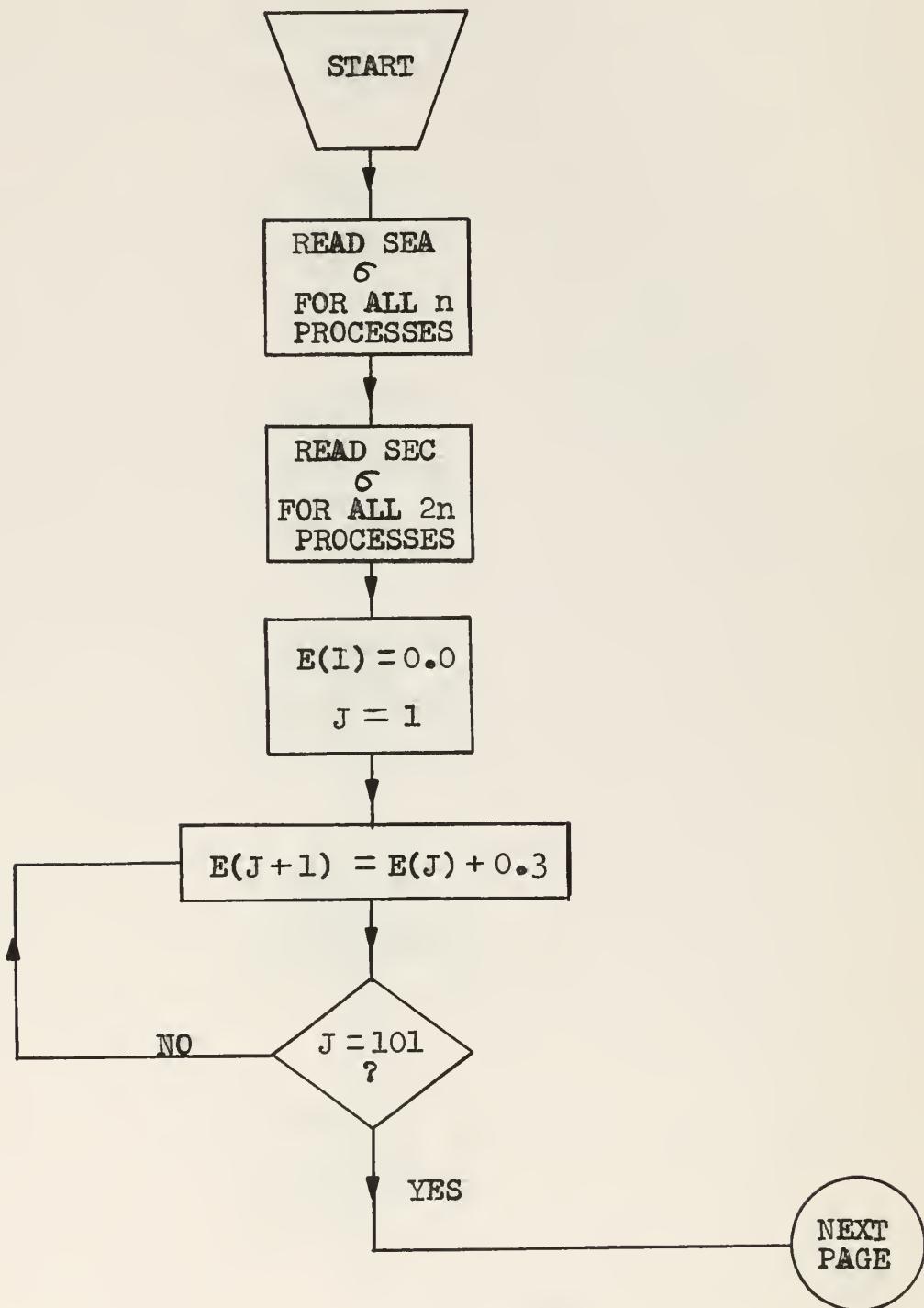
through NP6 will appear in the call statement as 2,2,1,1,2,1.

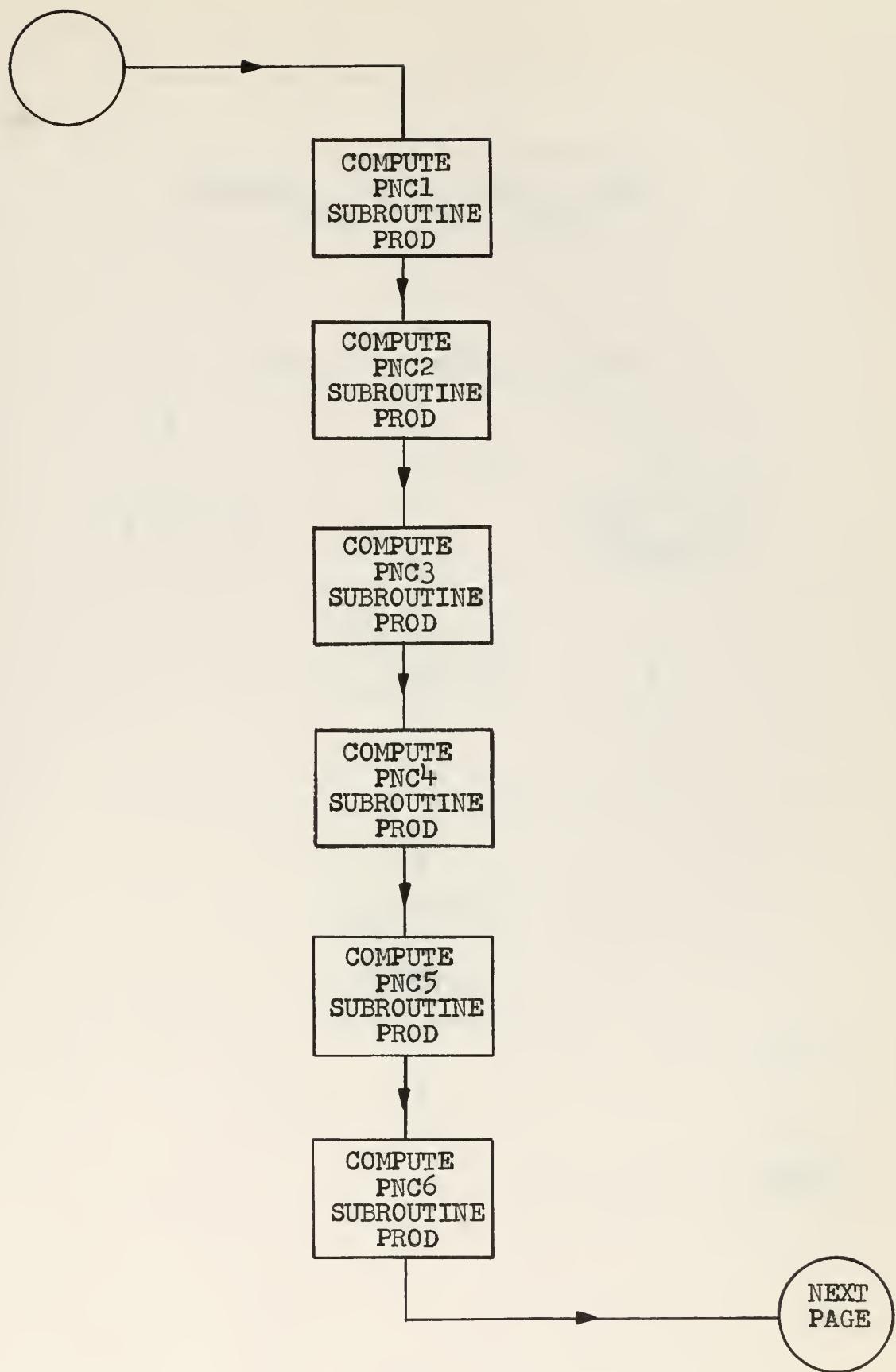
The value for NPT is the "leakage" factor. In this example NPT = Eth5/0.3.

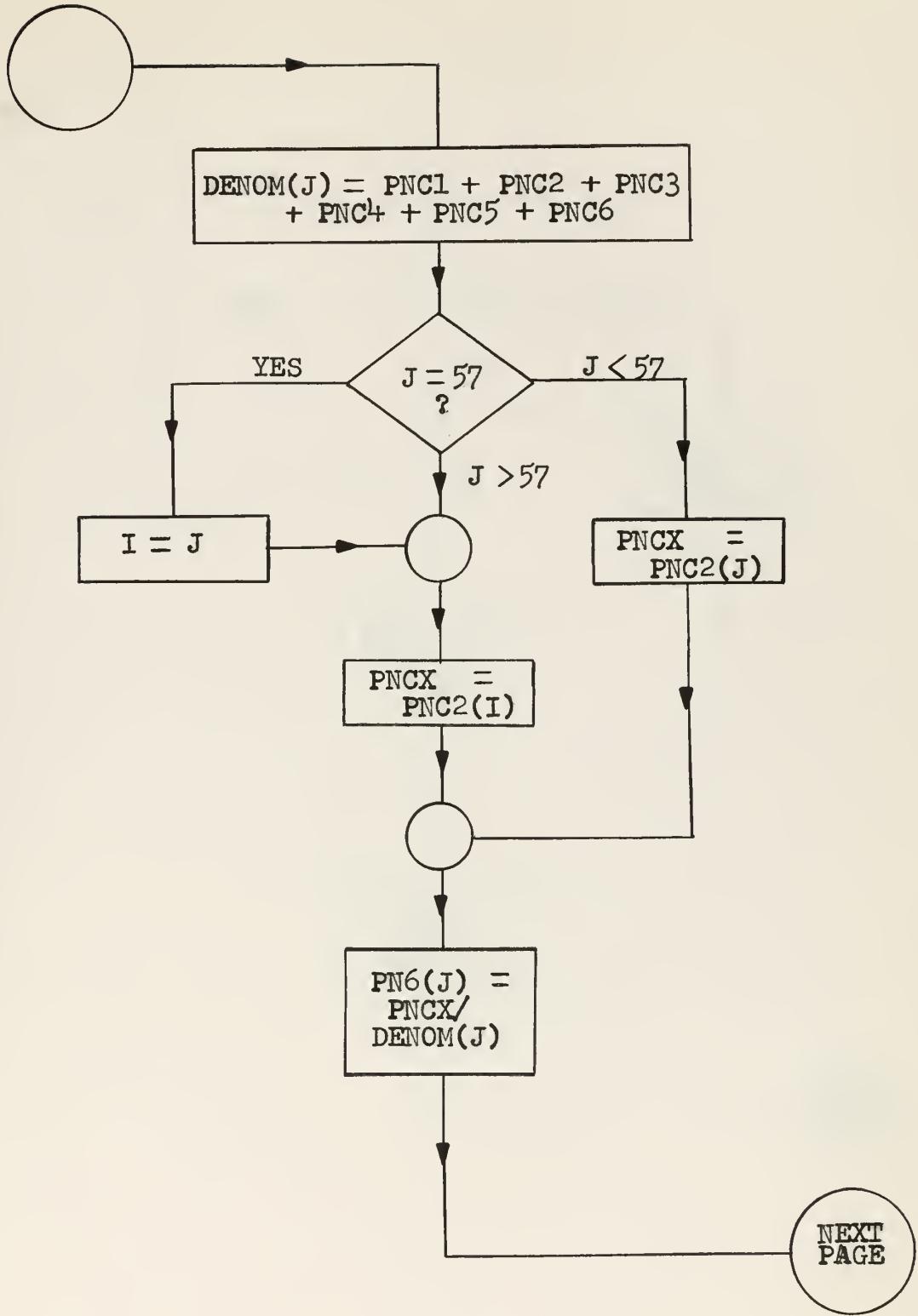
PN20 is the output of this subroutine and it represents the probability of the formation of cross-sections by a two-step process. PN20 may have any name in the call statement.

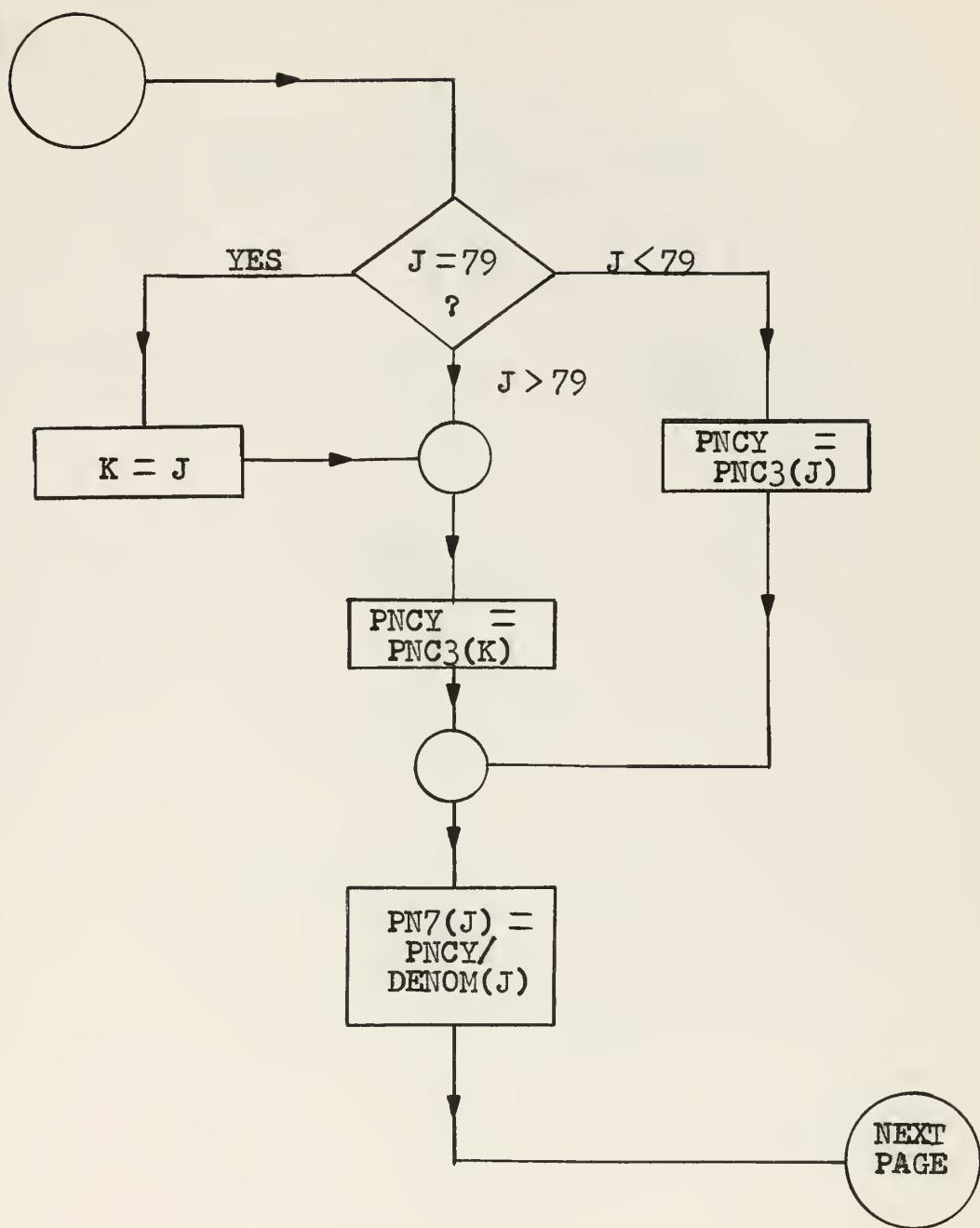
The triple integral subroutine follows the same general procedure.

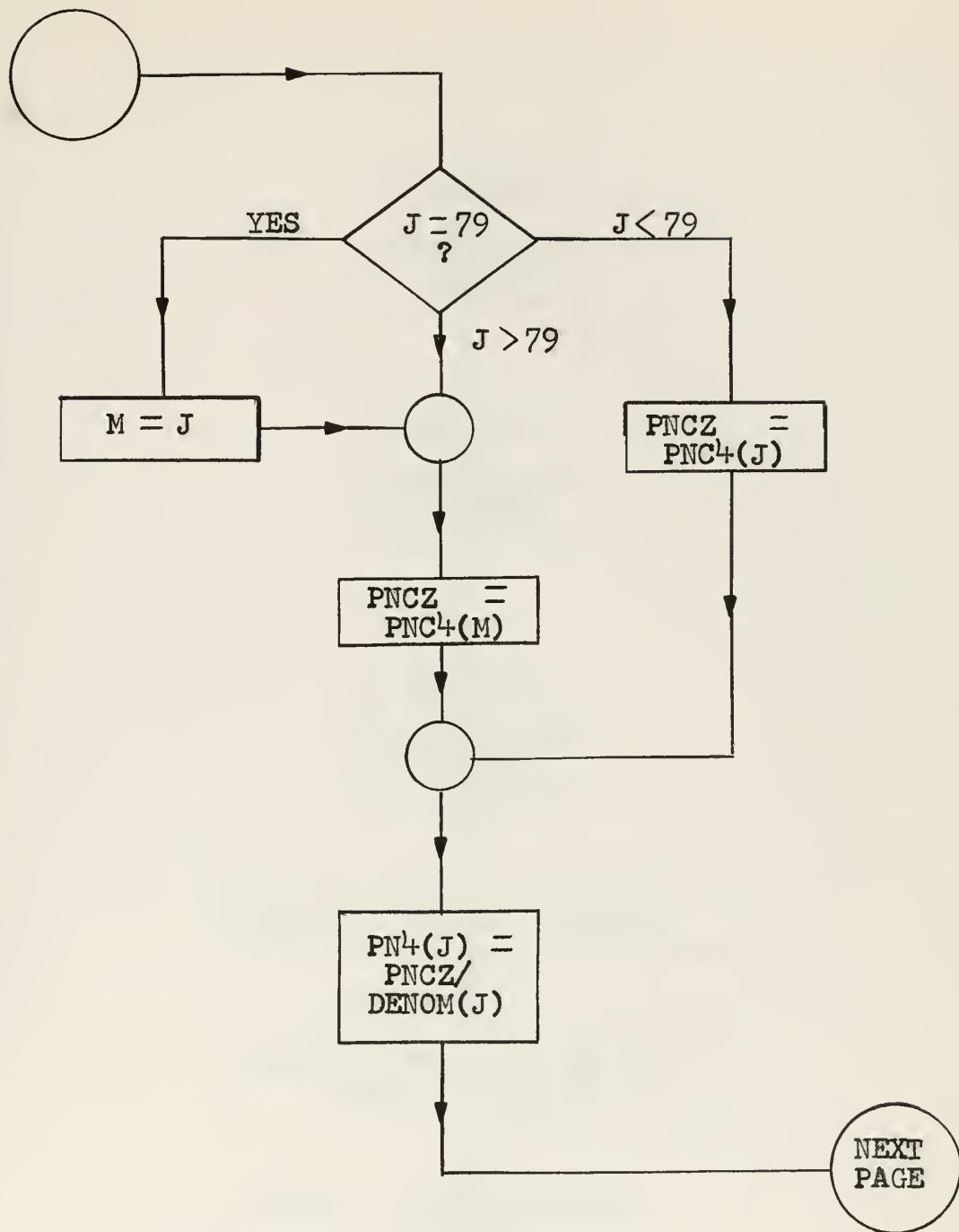
APPENDIX II
PROGRAM FLOW CHART

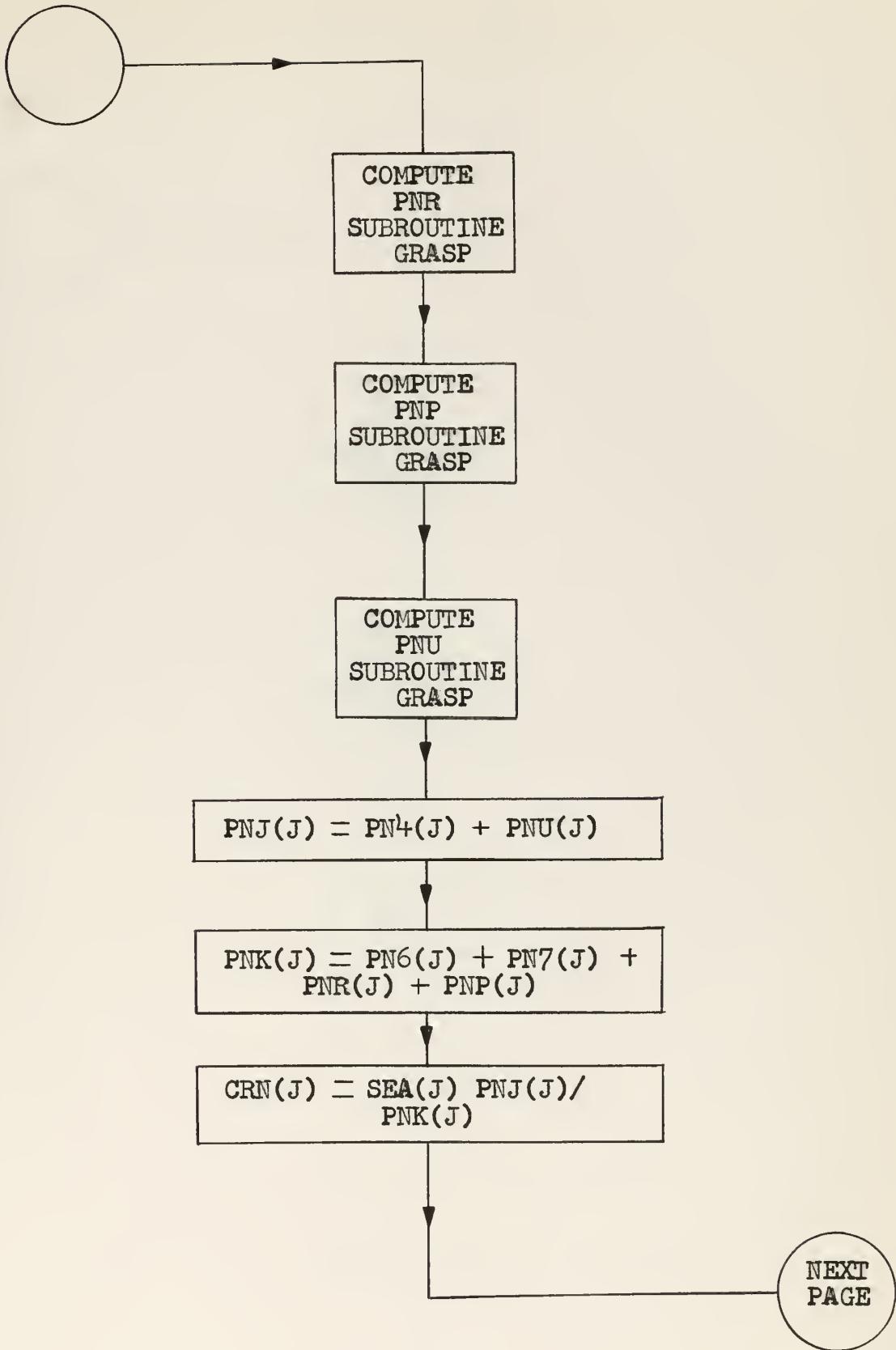


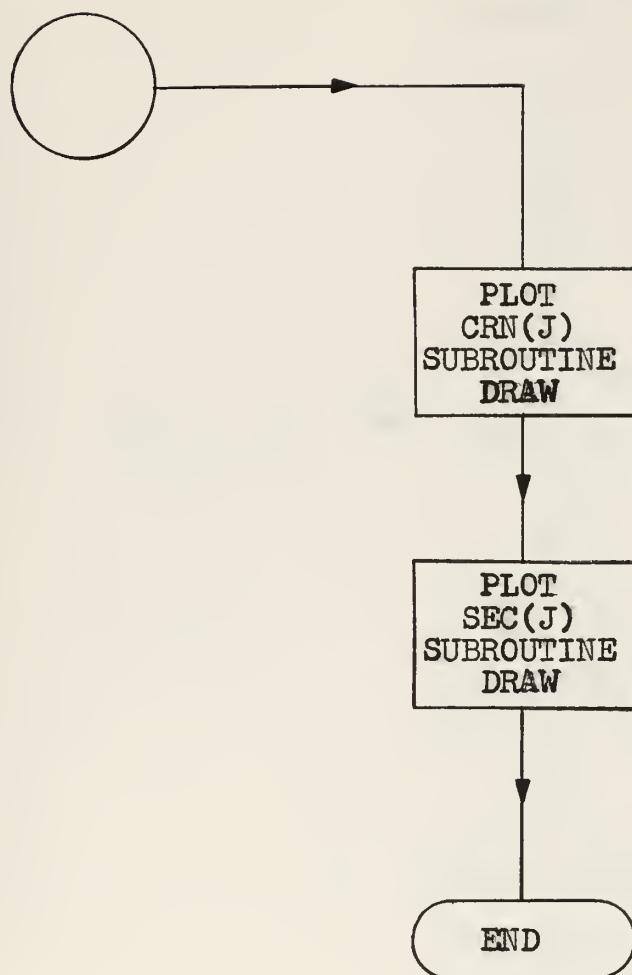




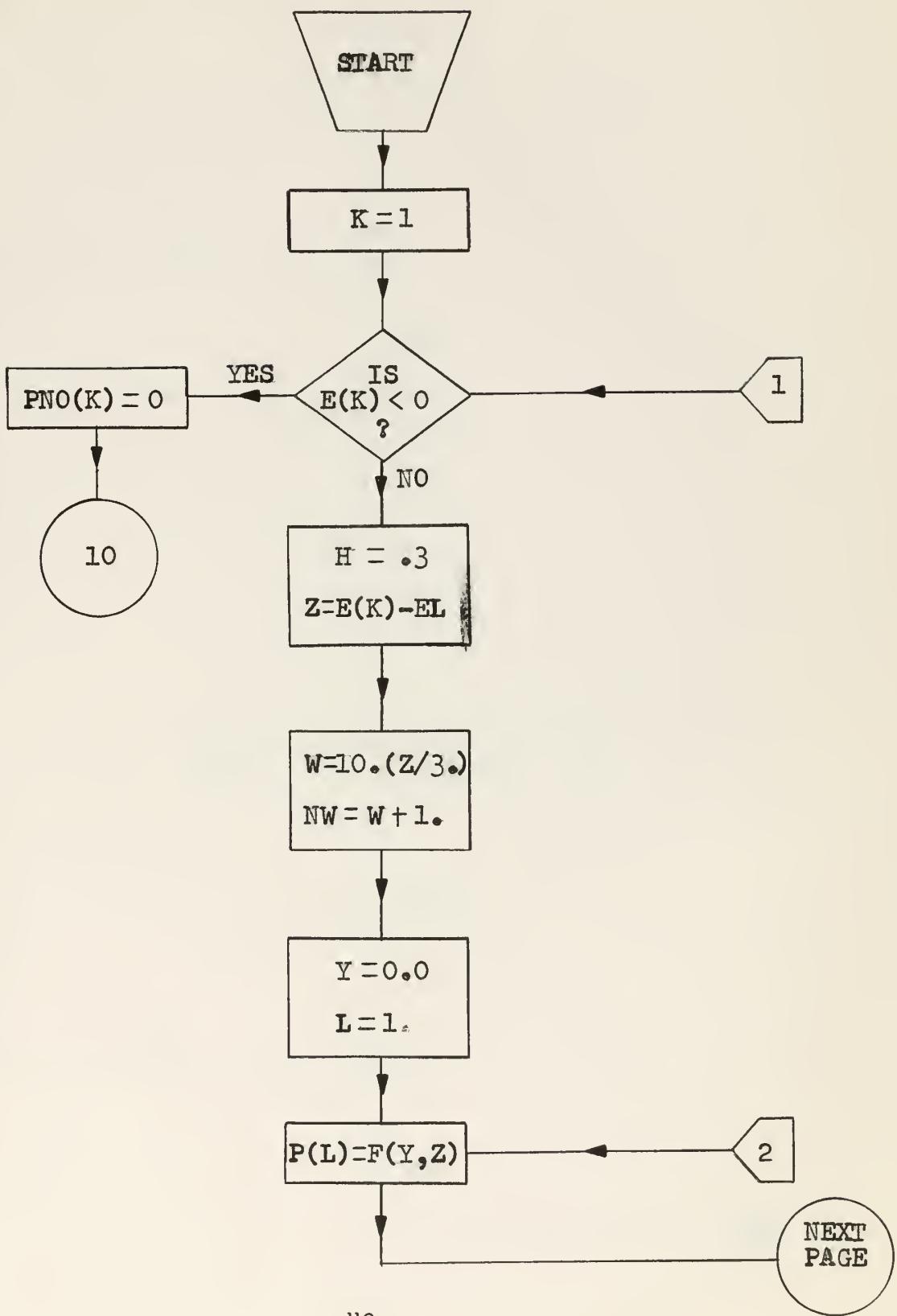


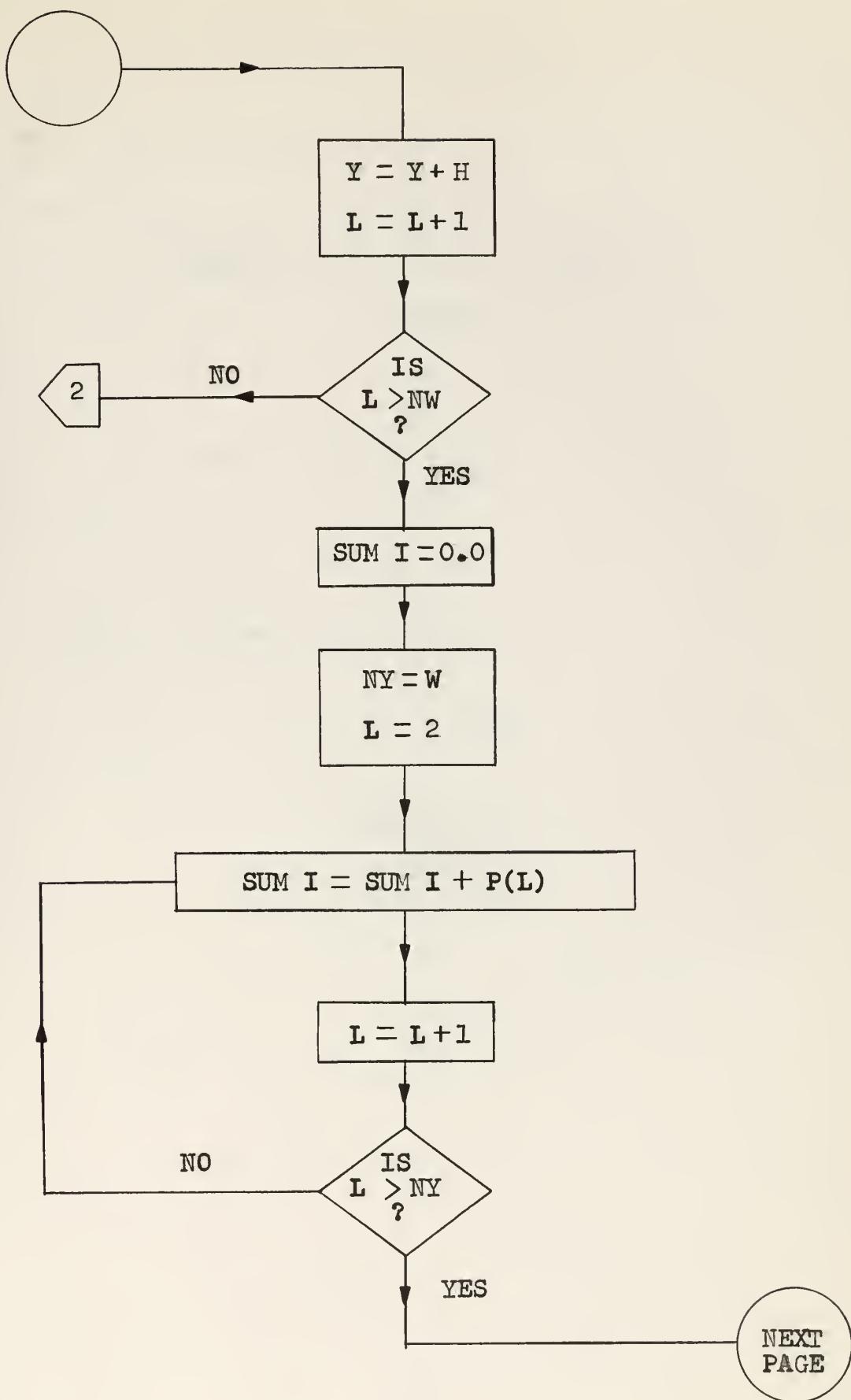


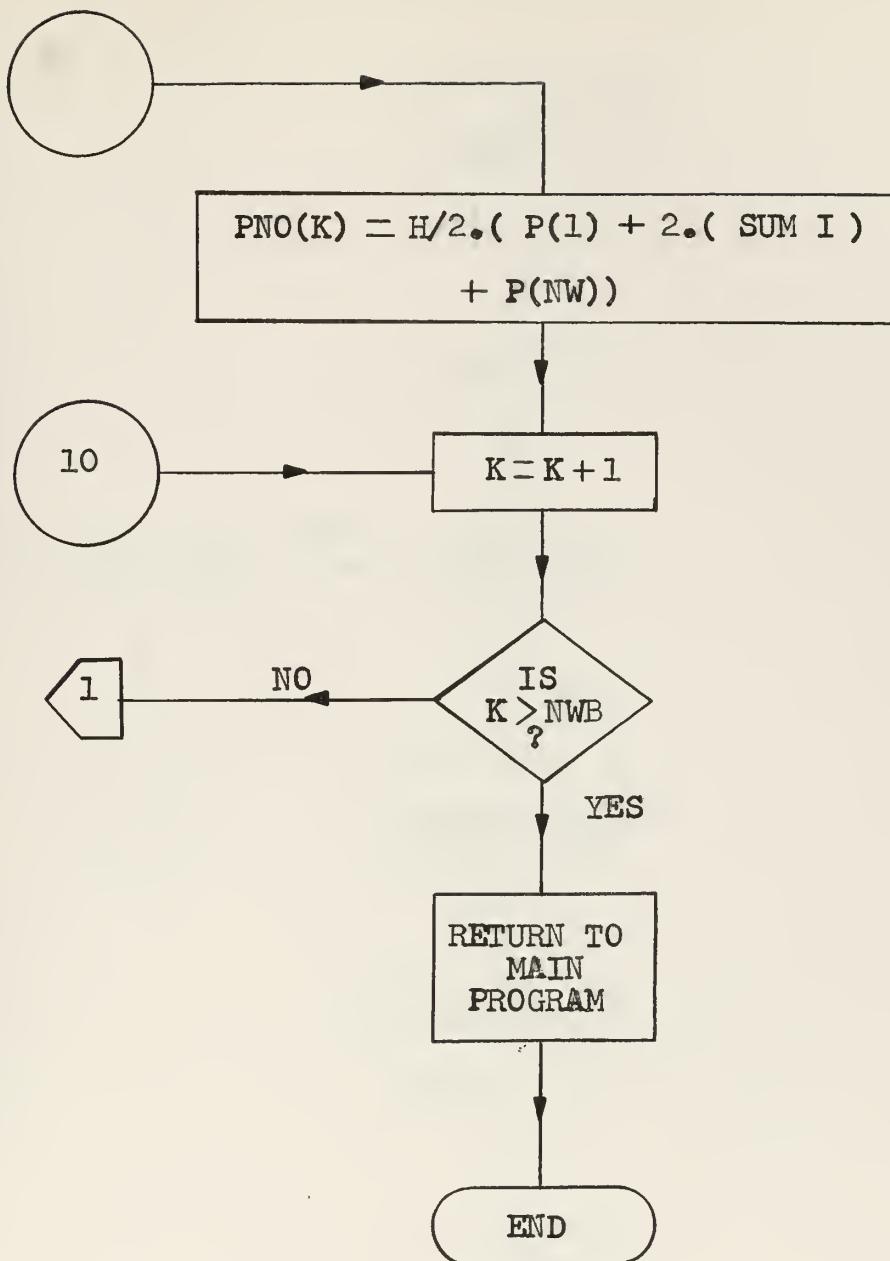




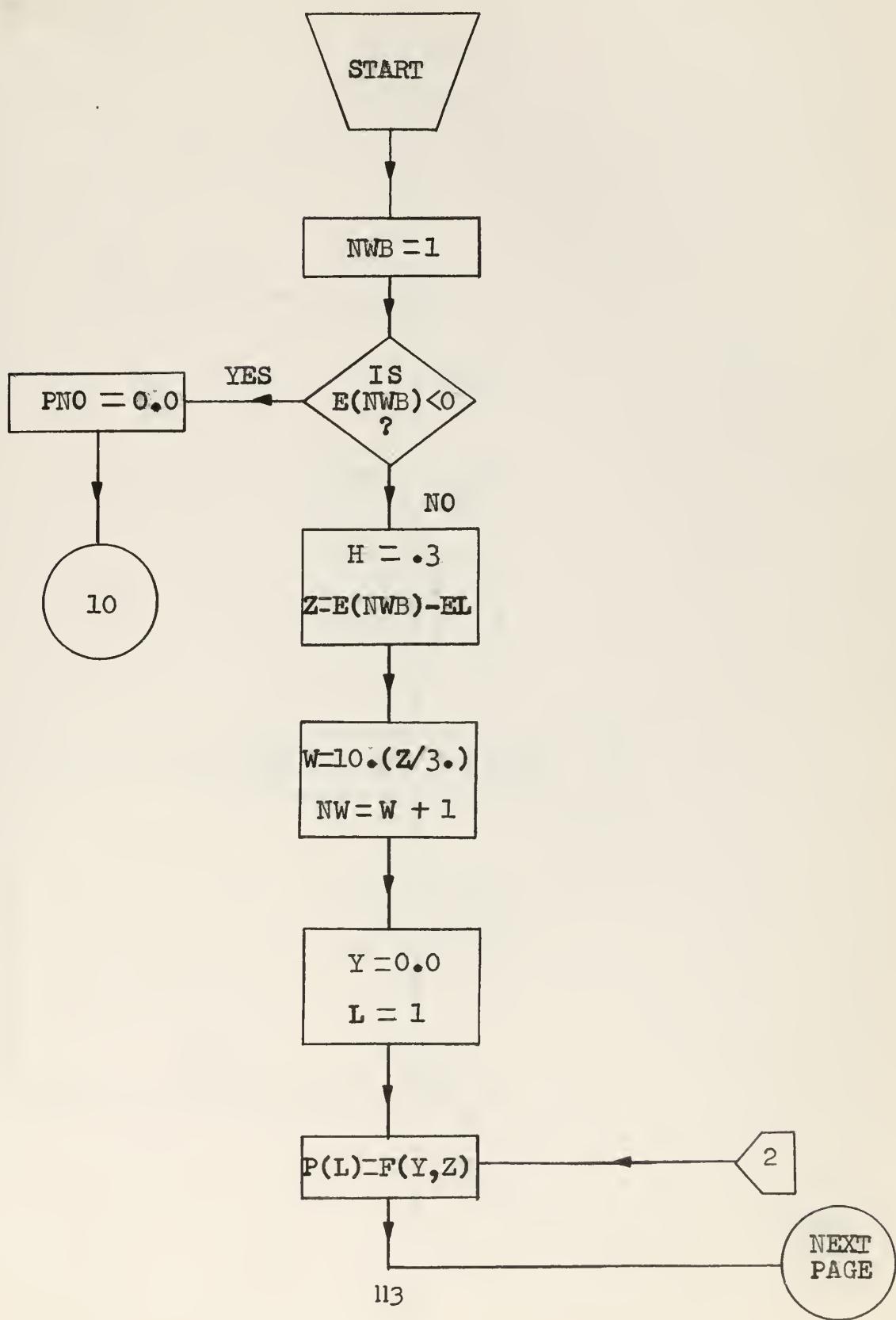
SUBROUTINE PROD

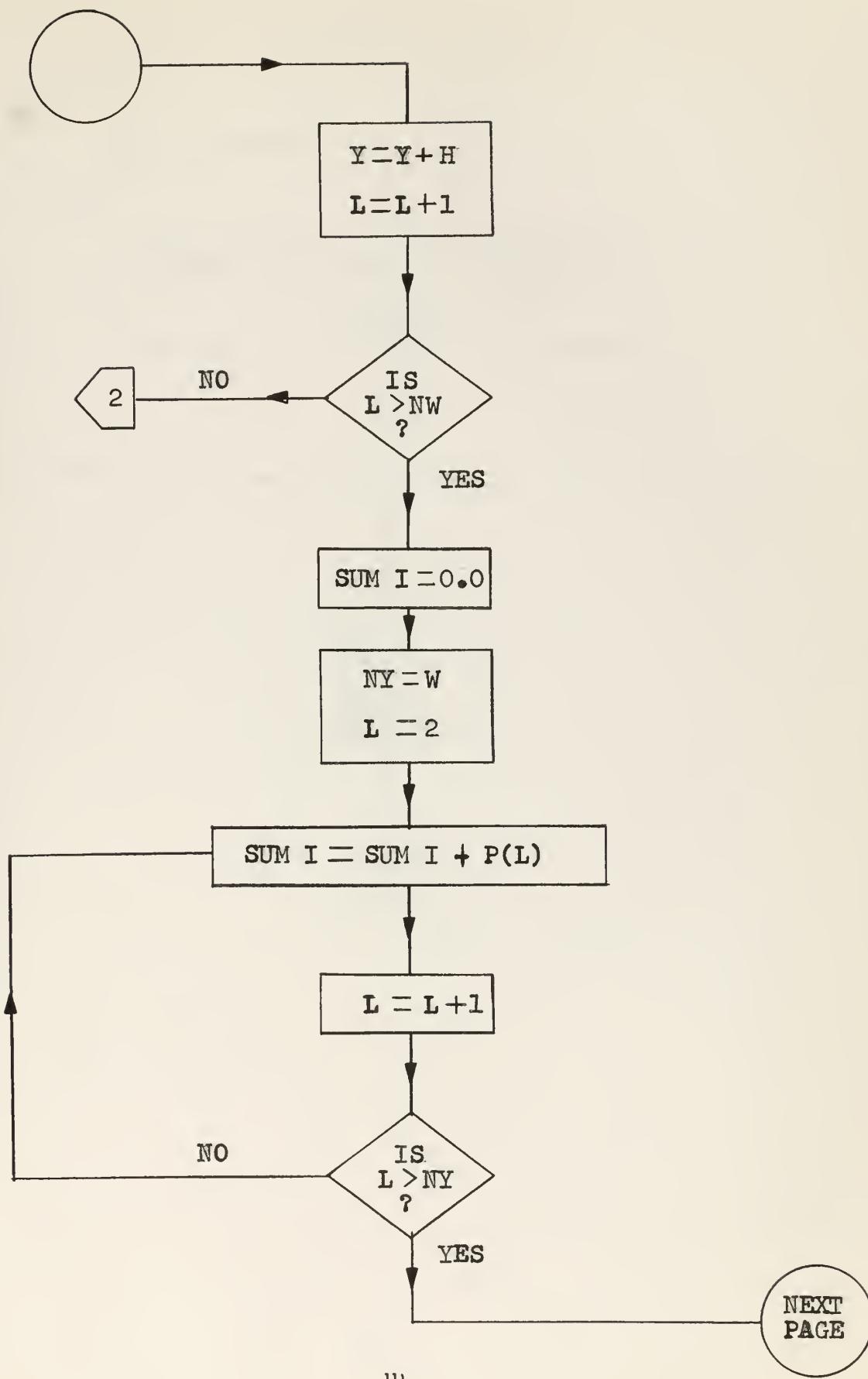


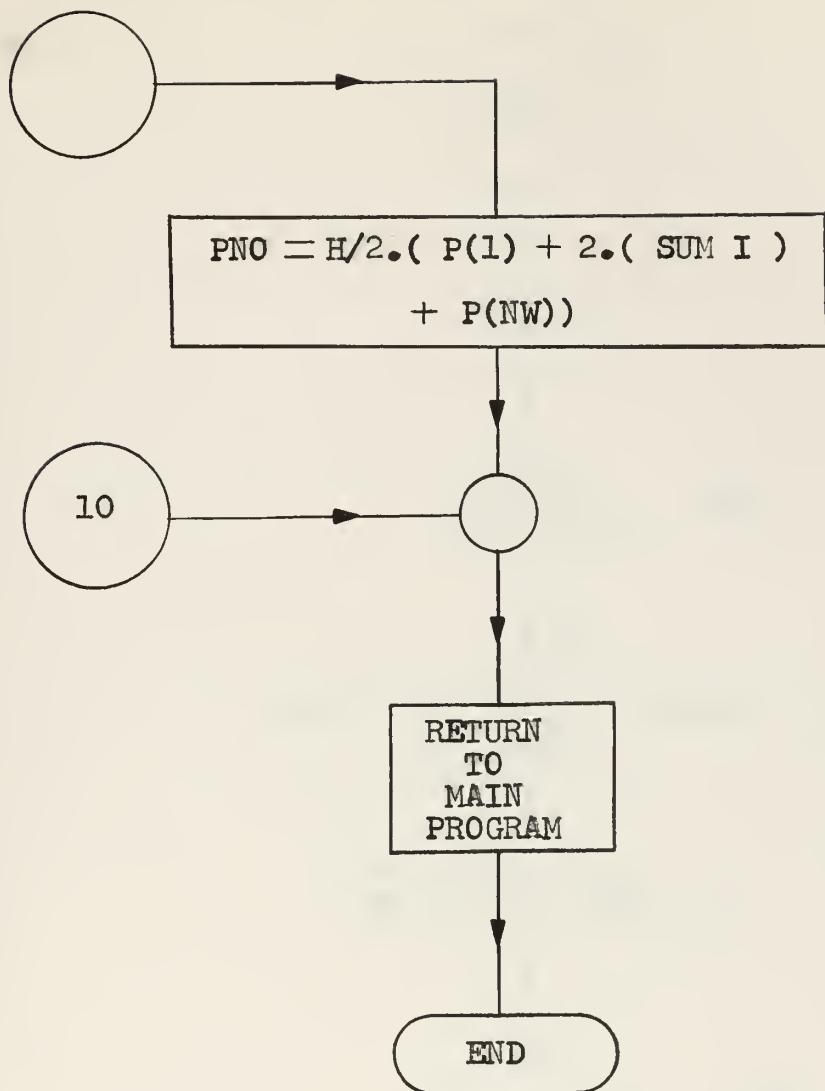




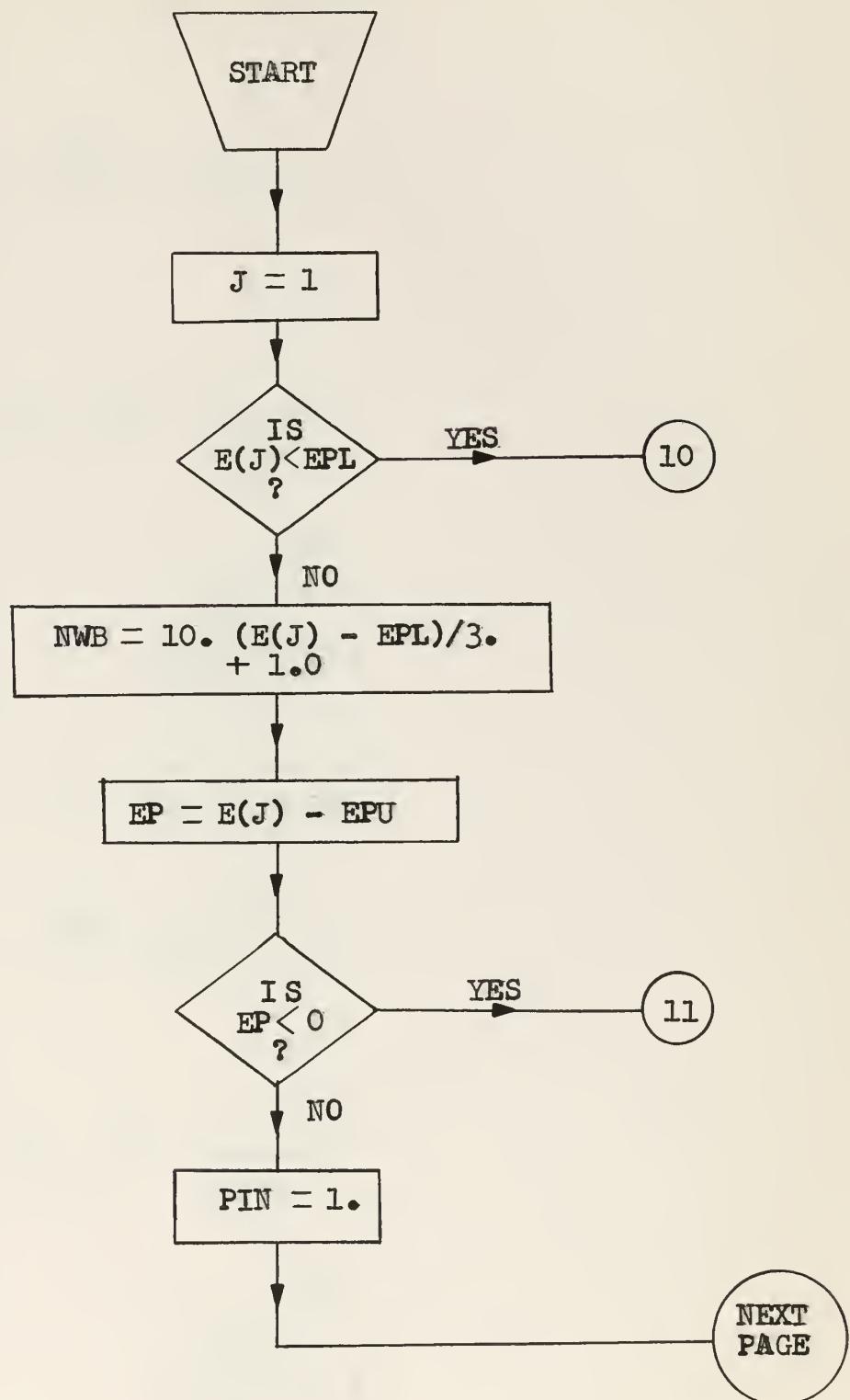
SUBROUTINE PROBE II

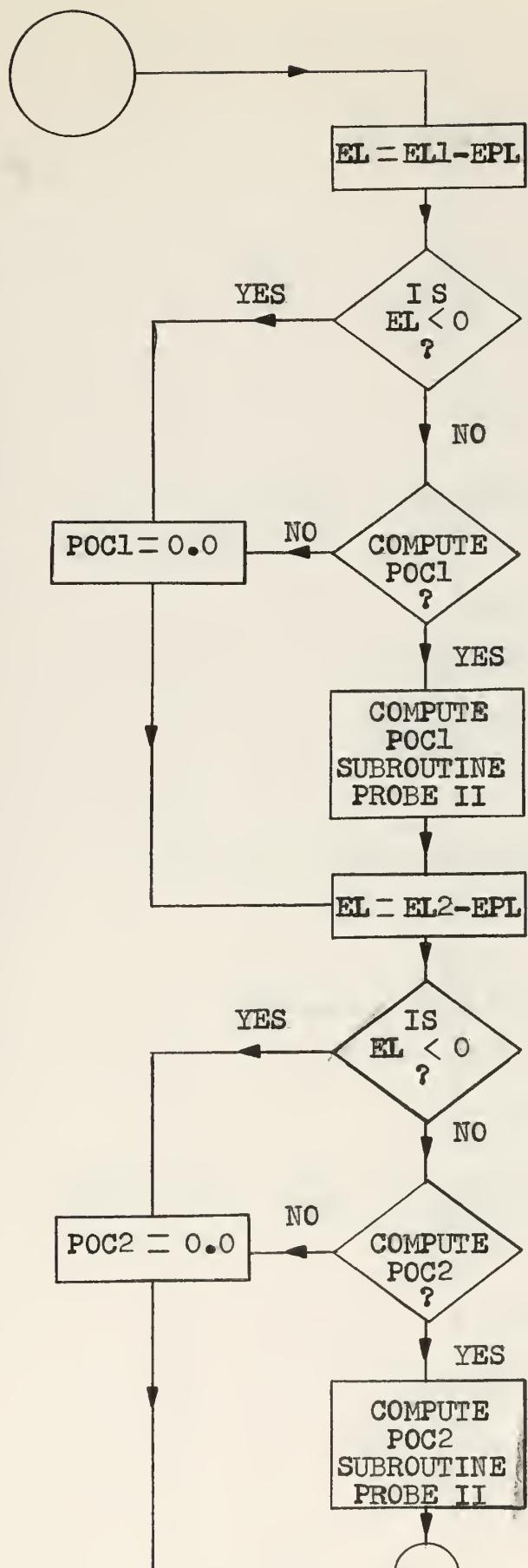


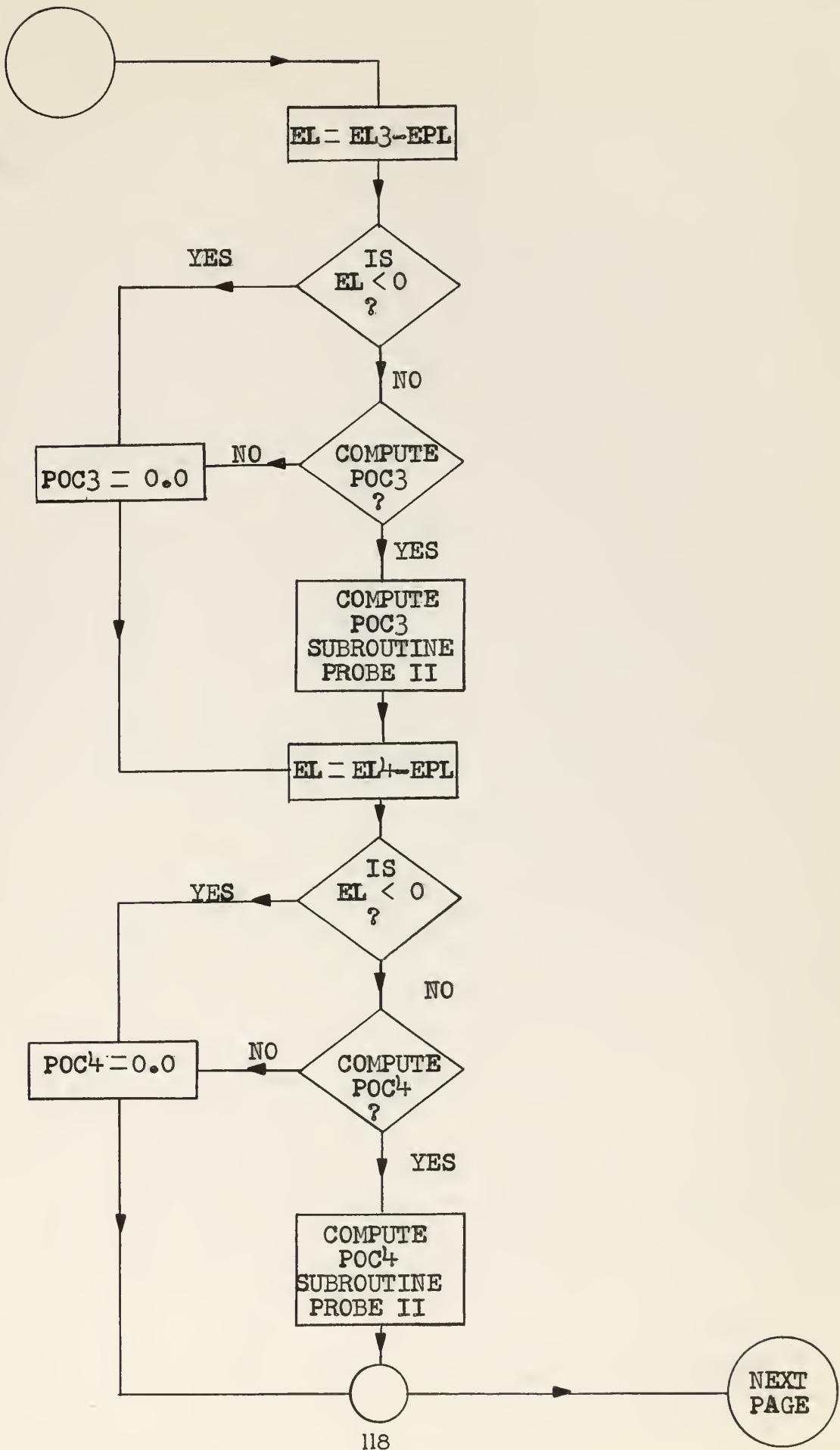


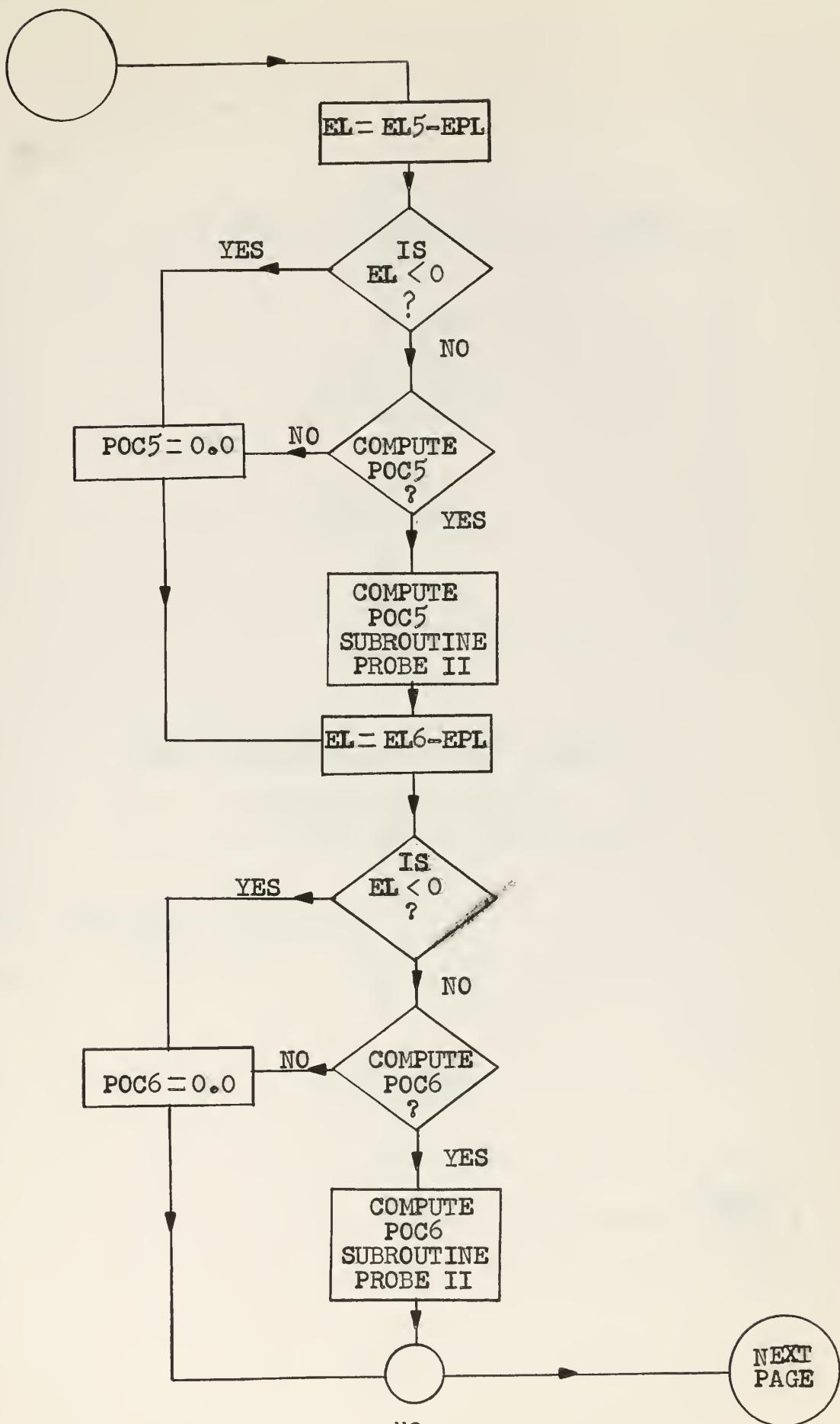


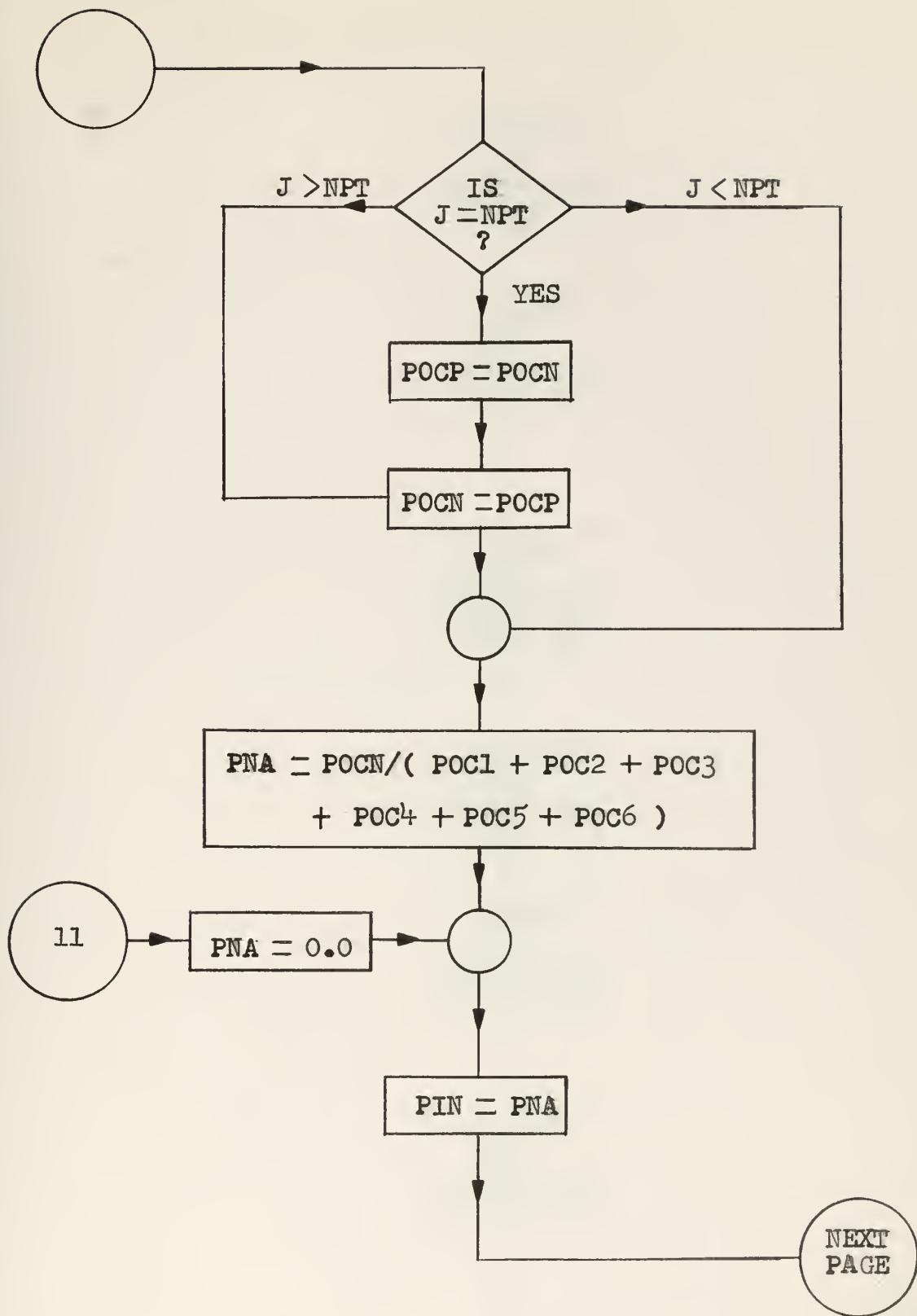
SUBROUTINE GRASP

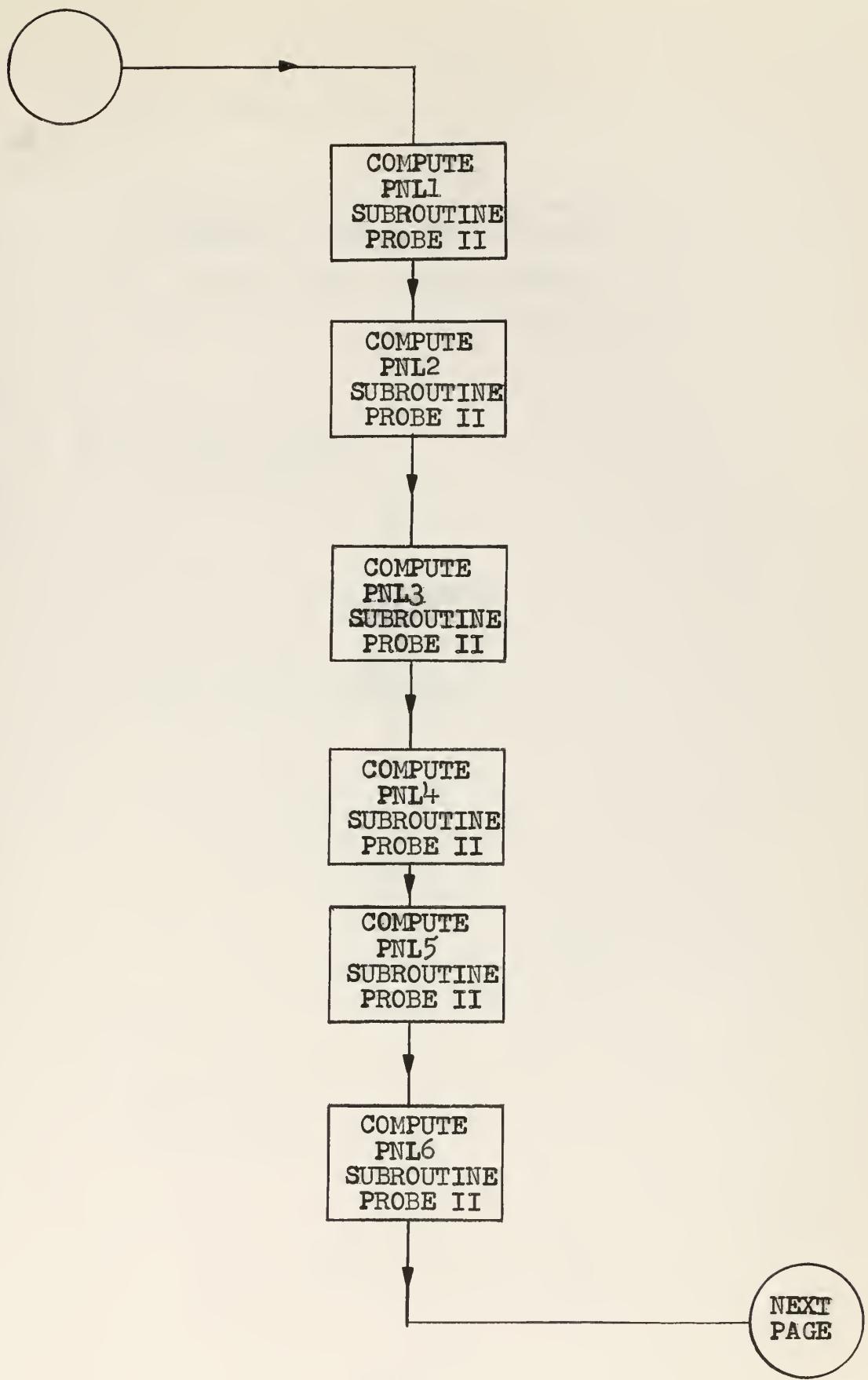


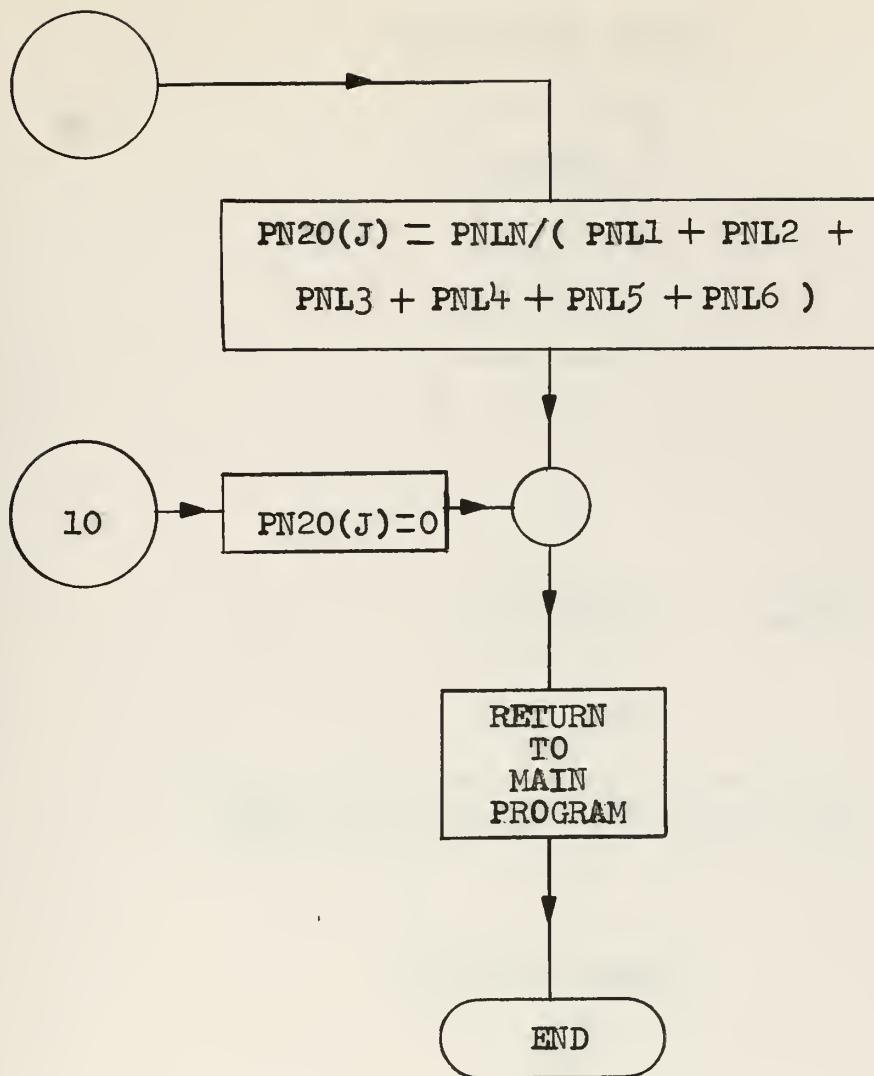






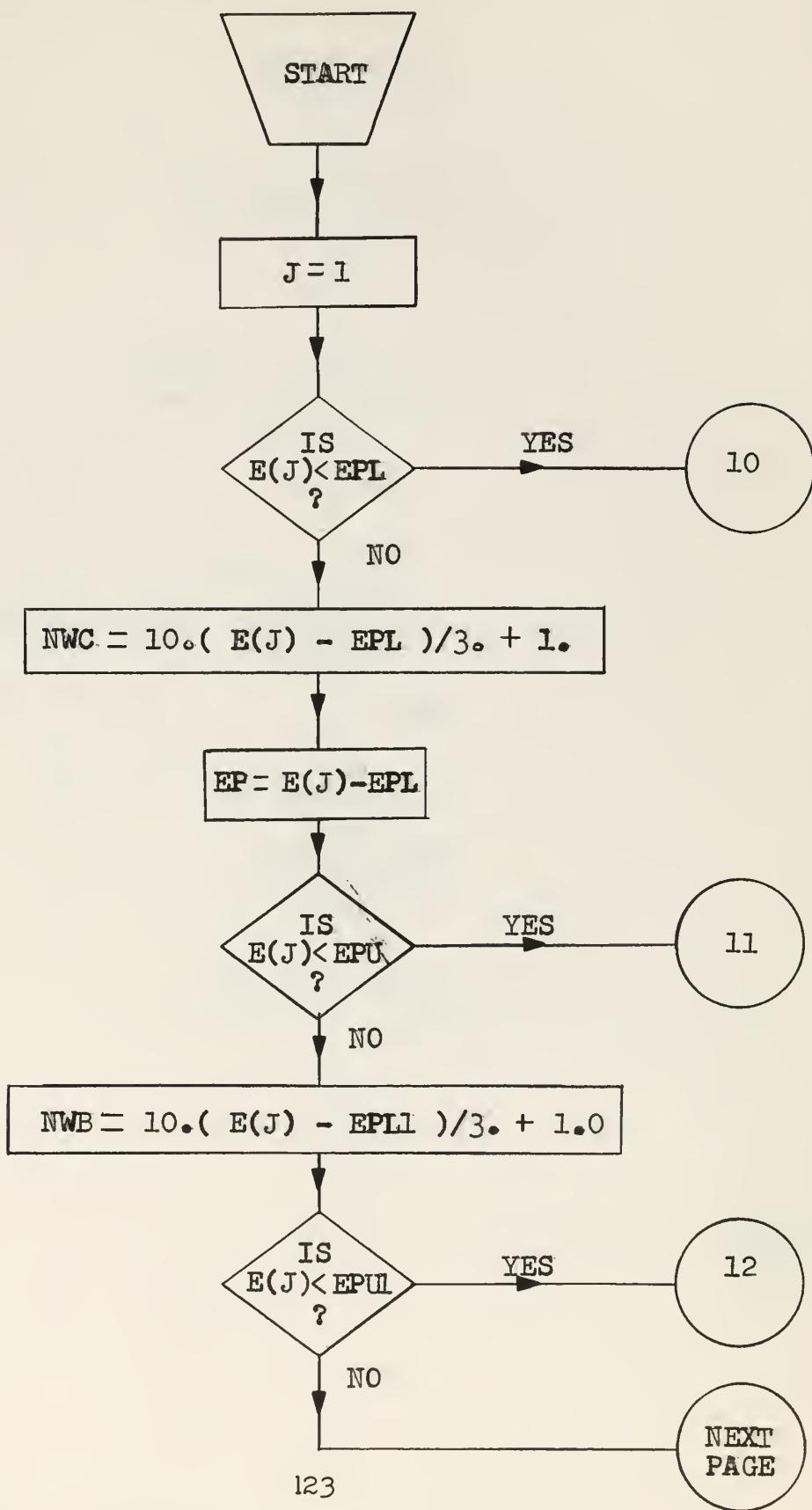


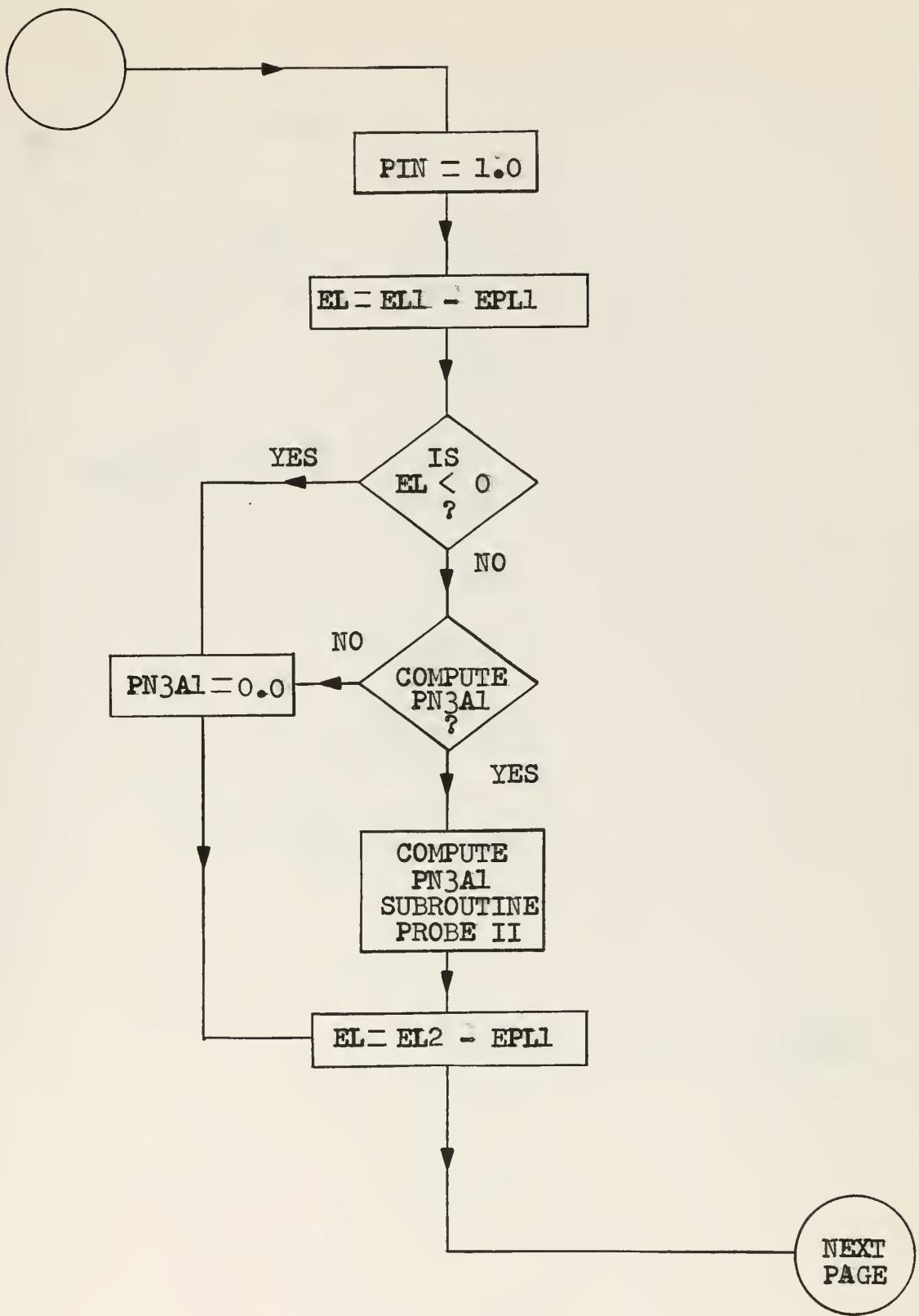


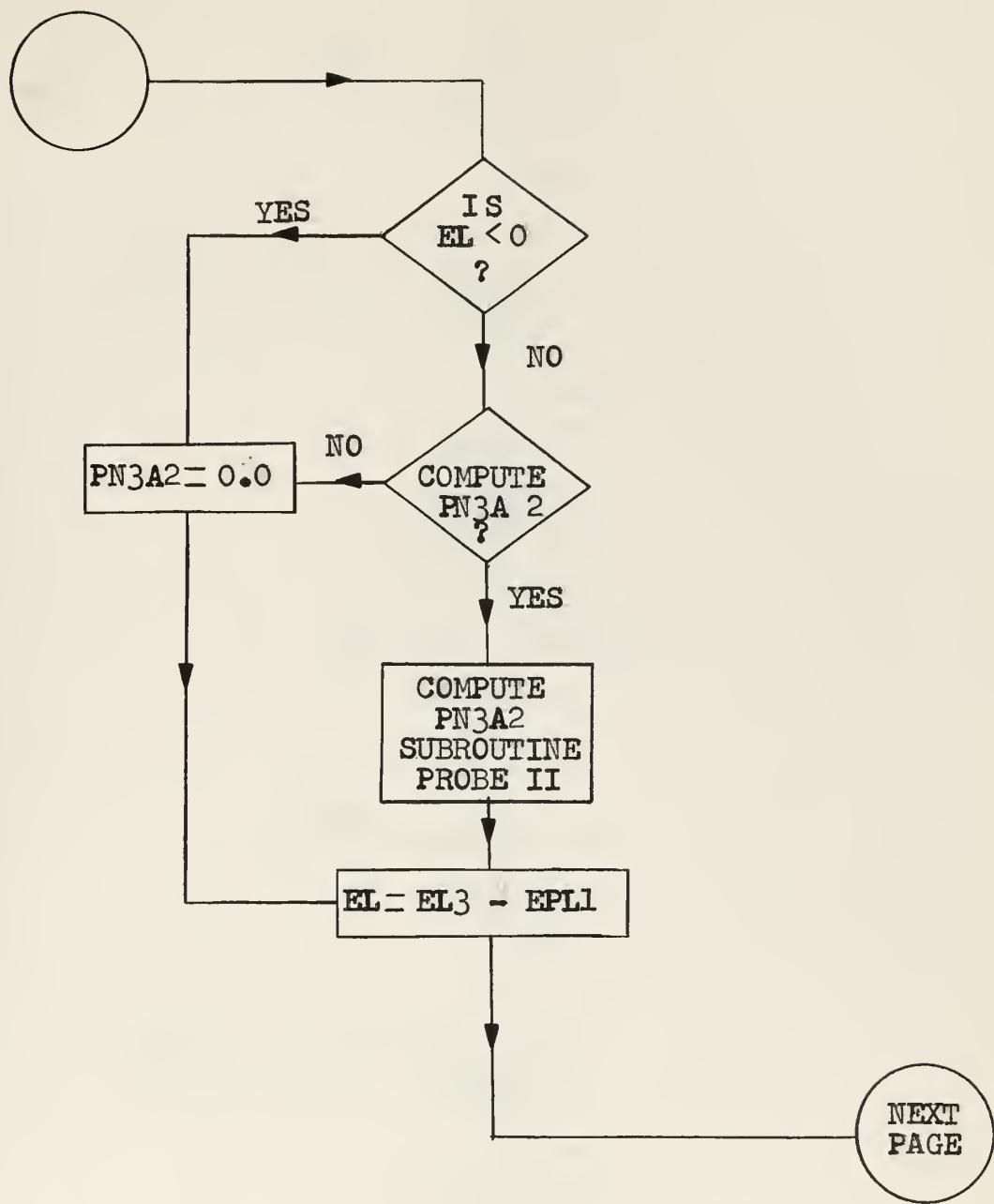


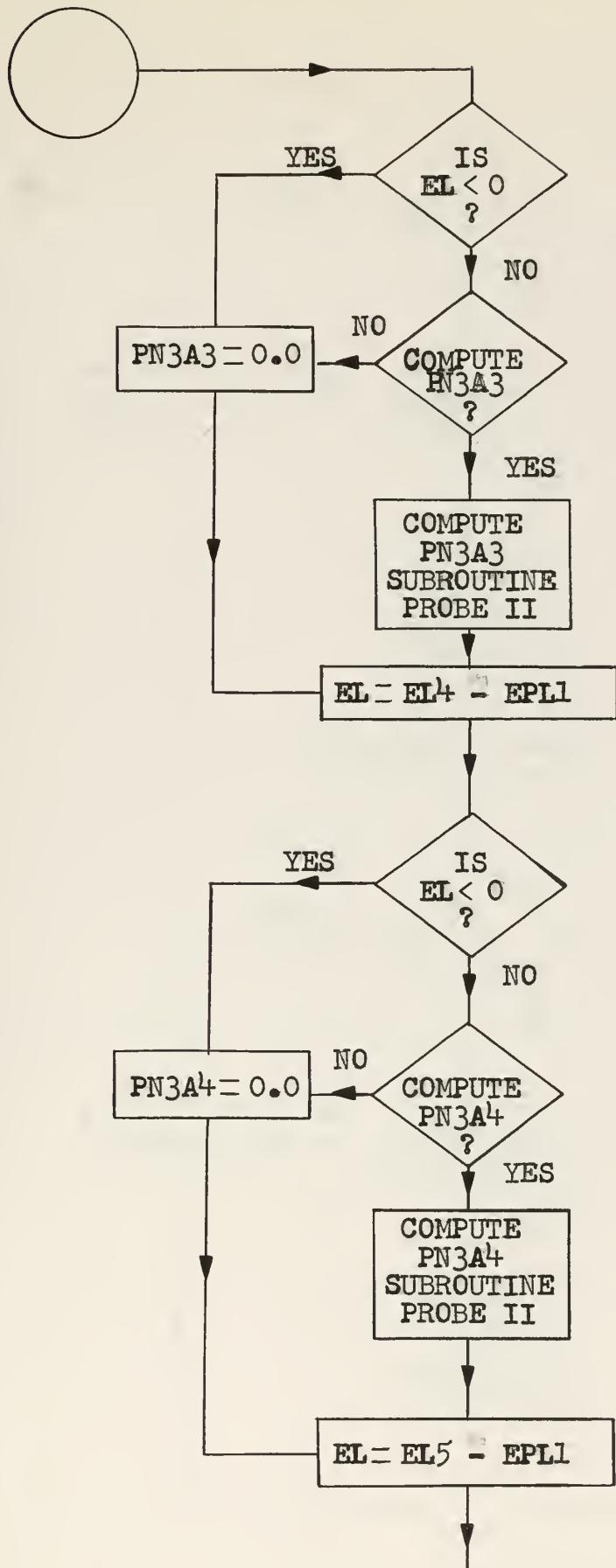
APPENDIX IIA

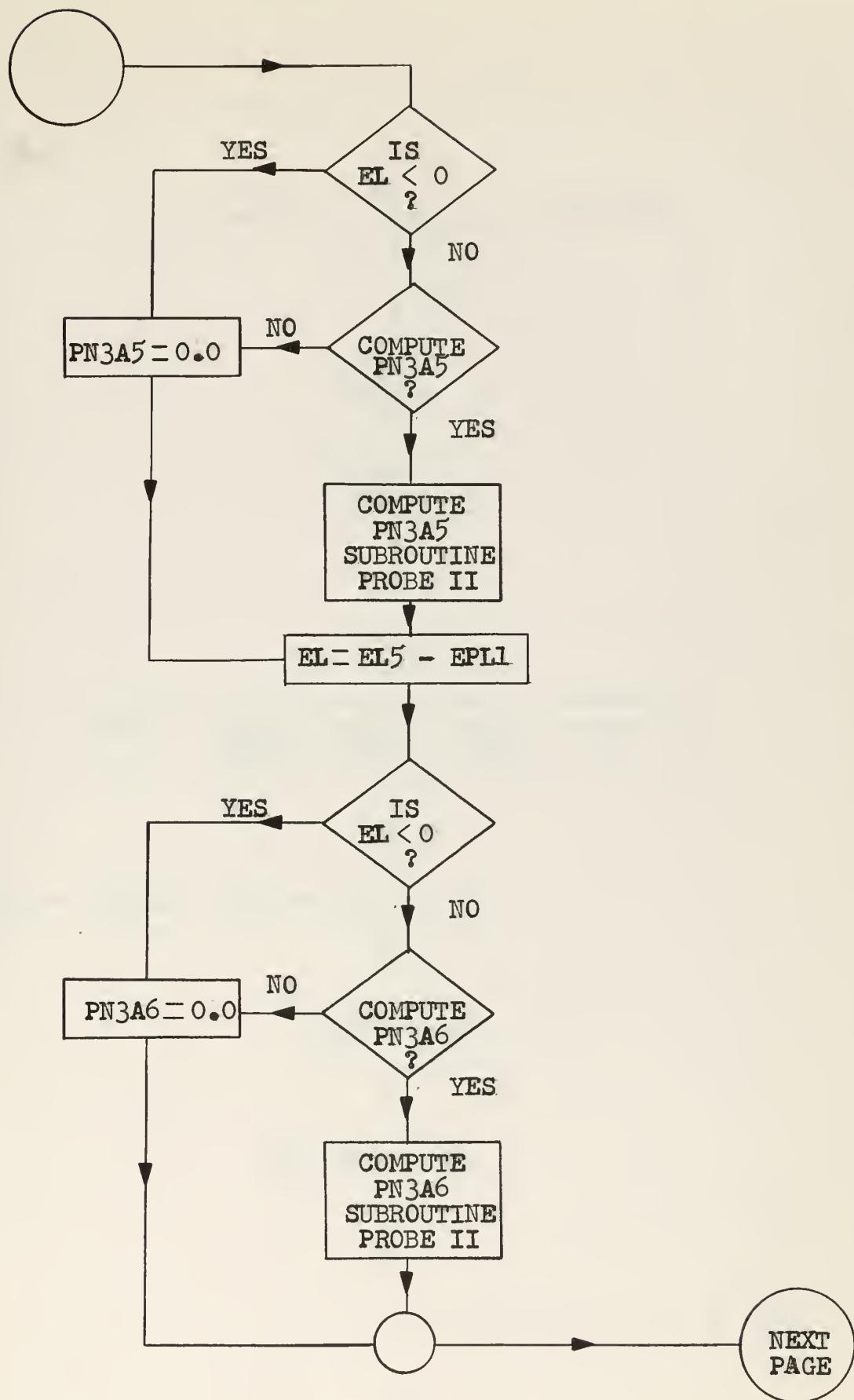
SUBROUTINE GROPE

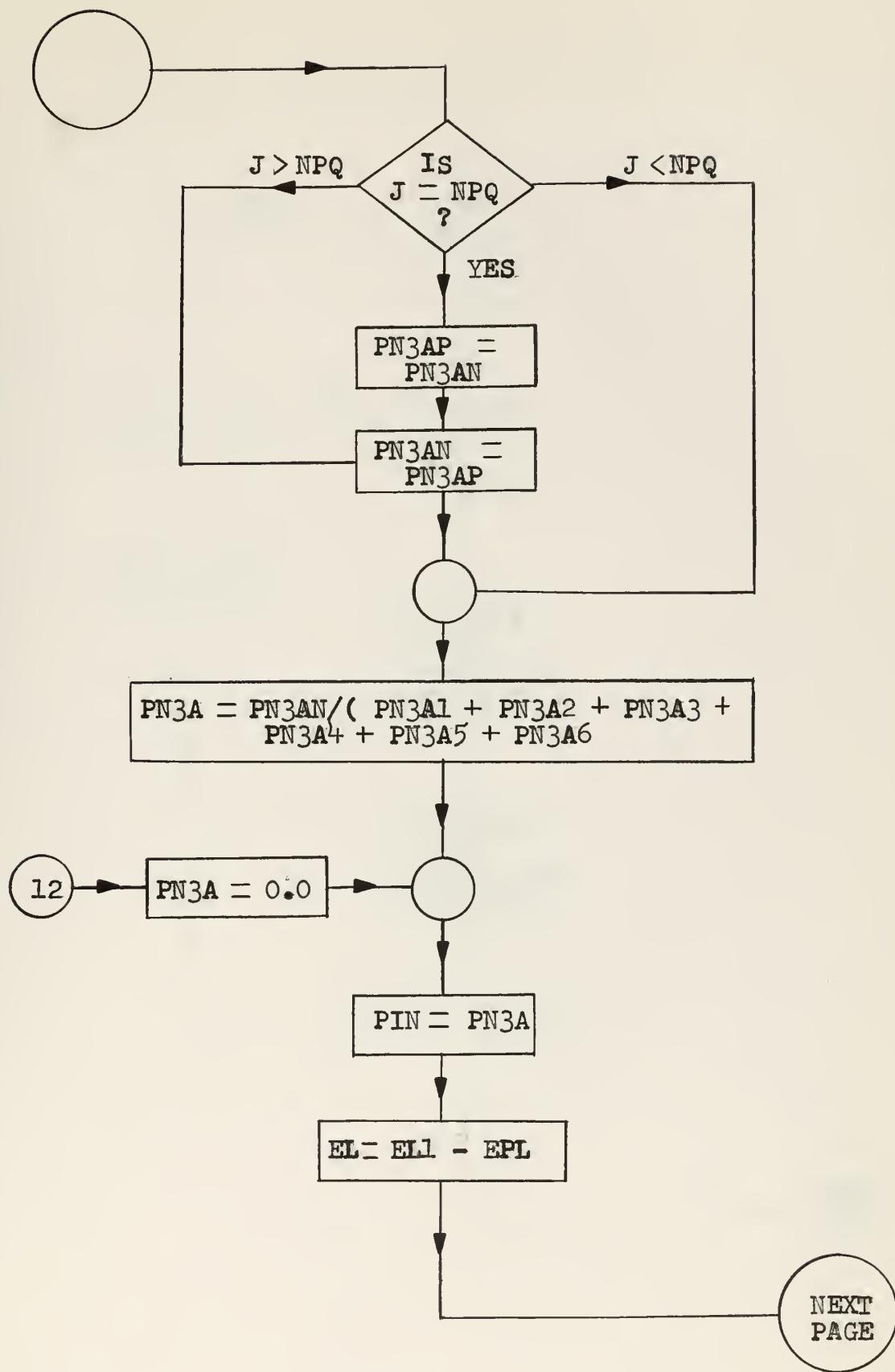


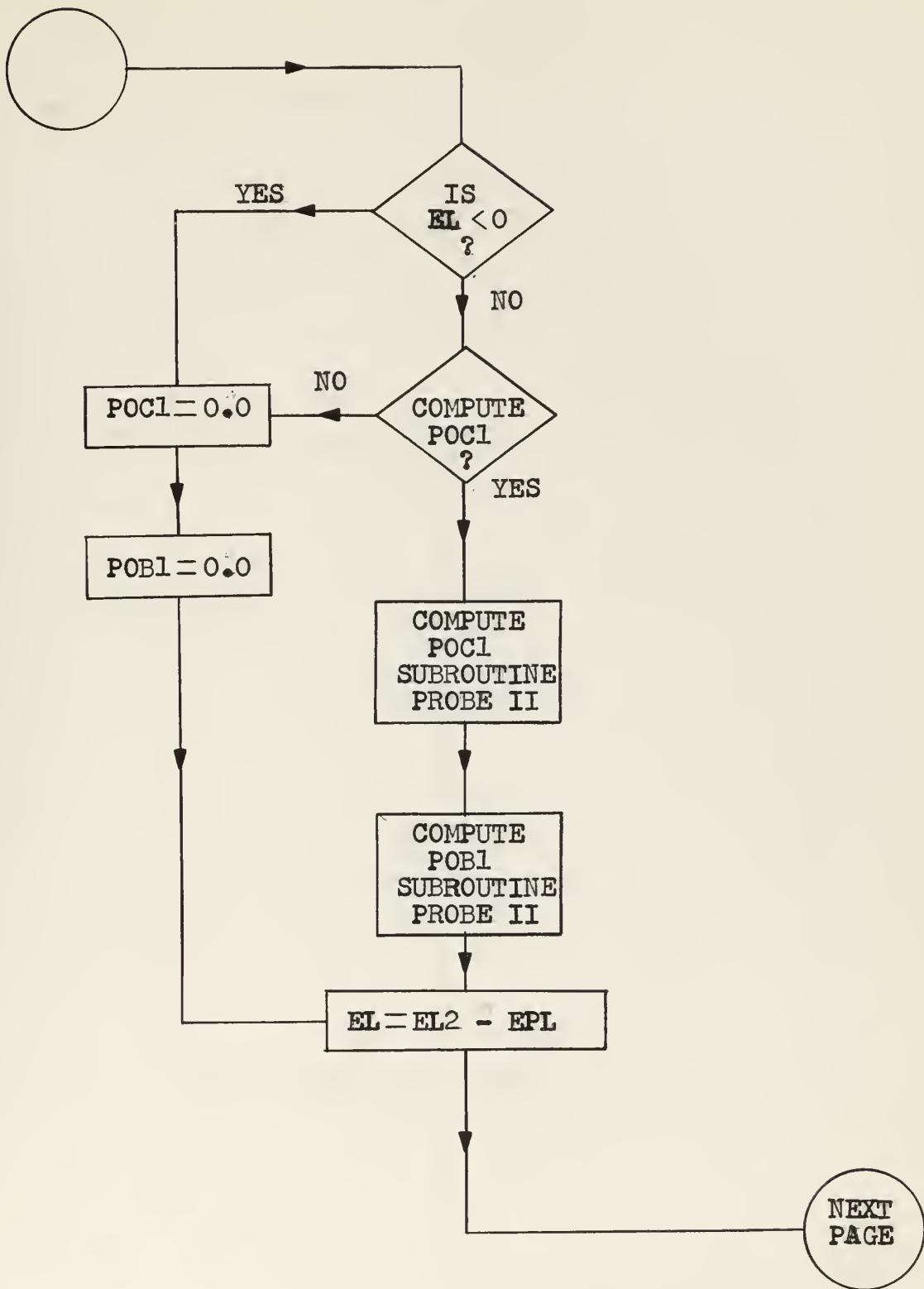


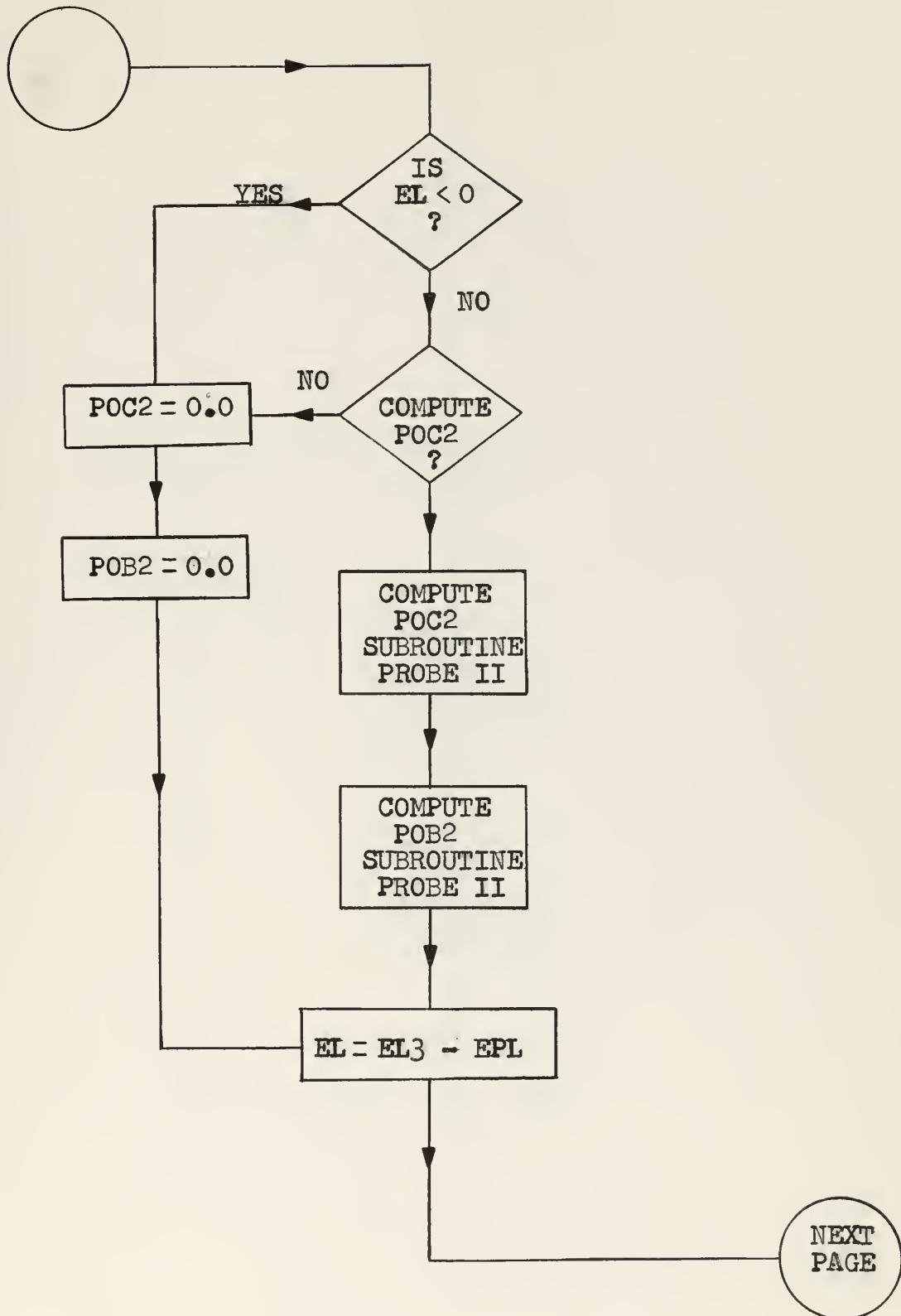


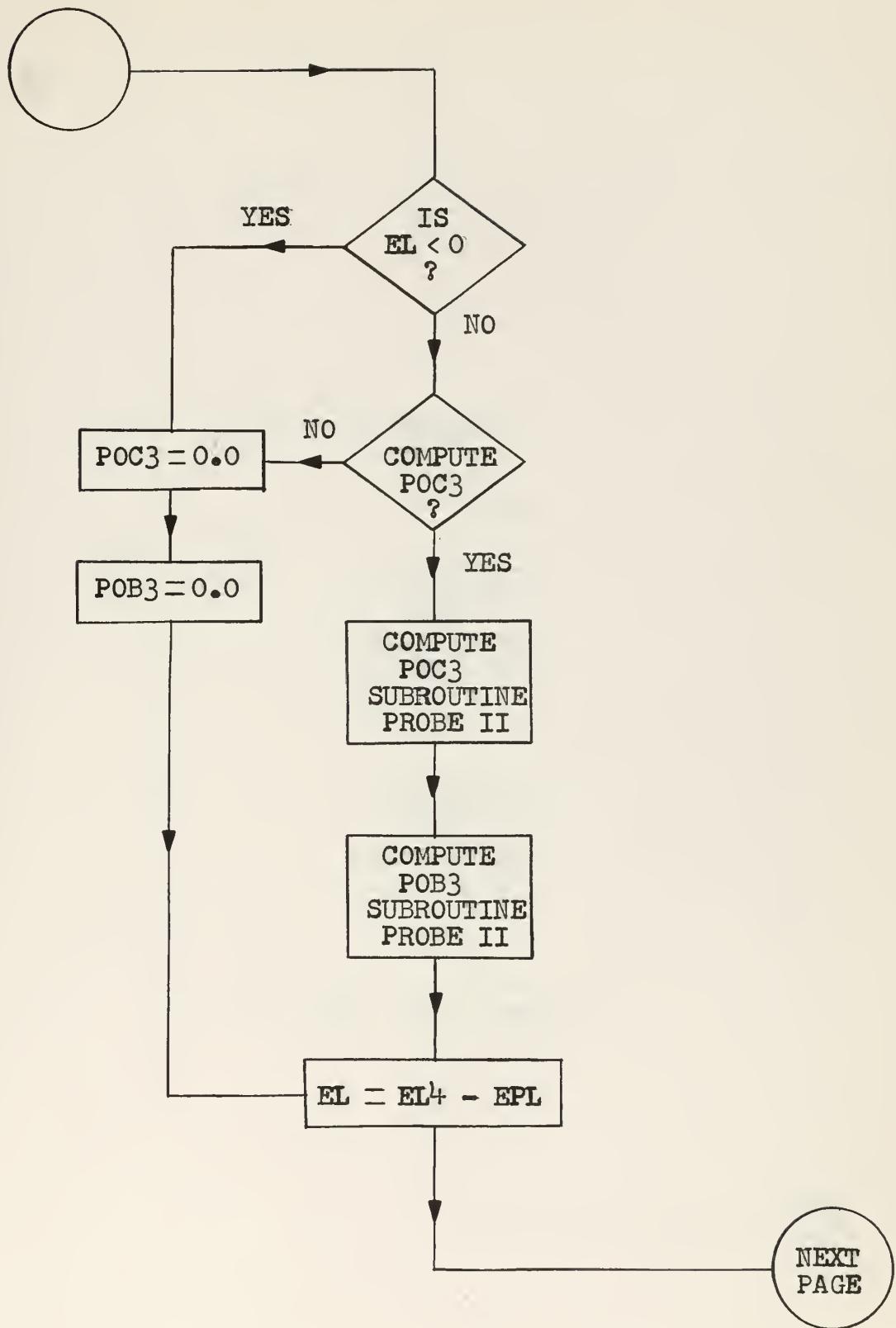


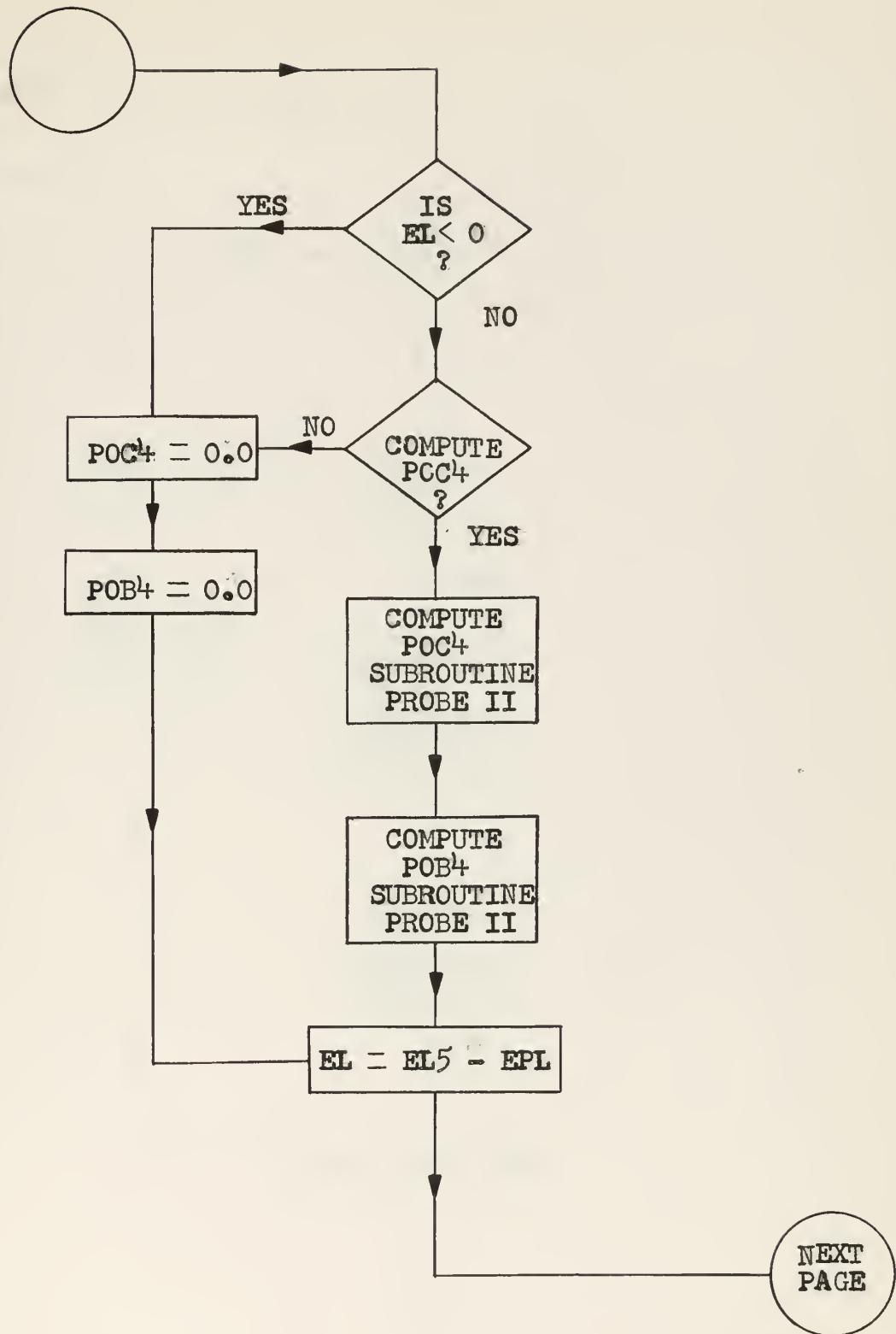


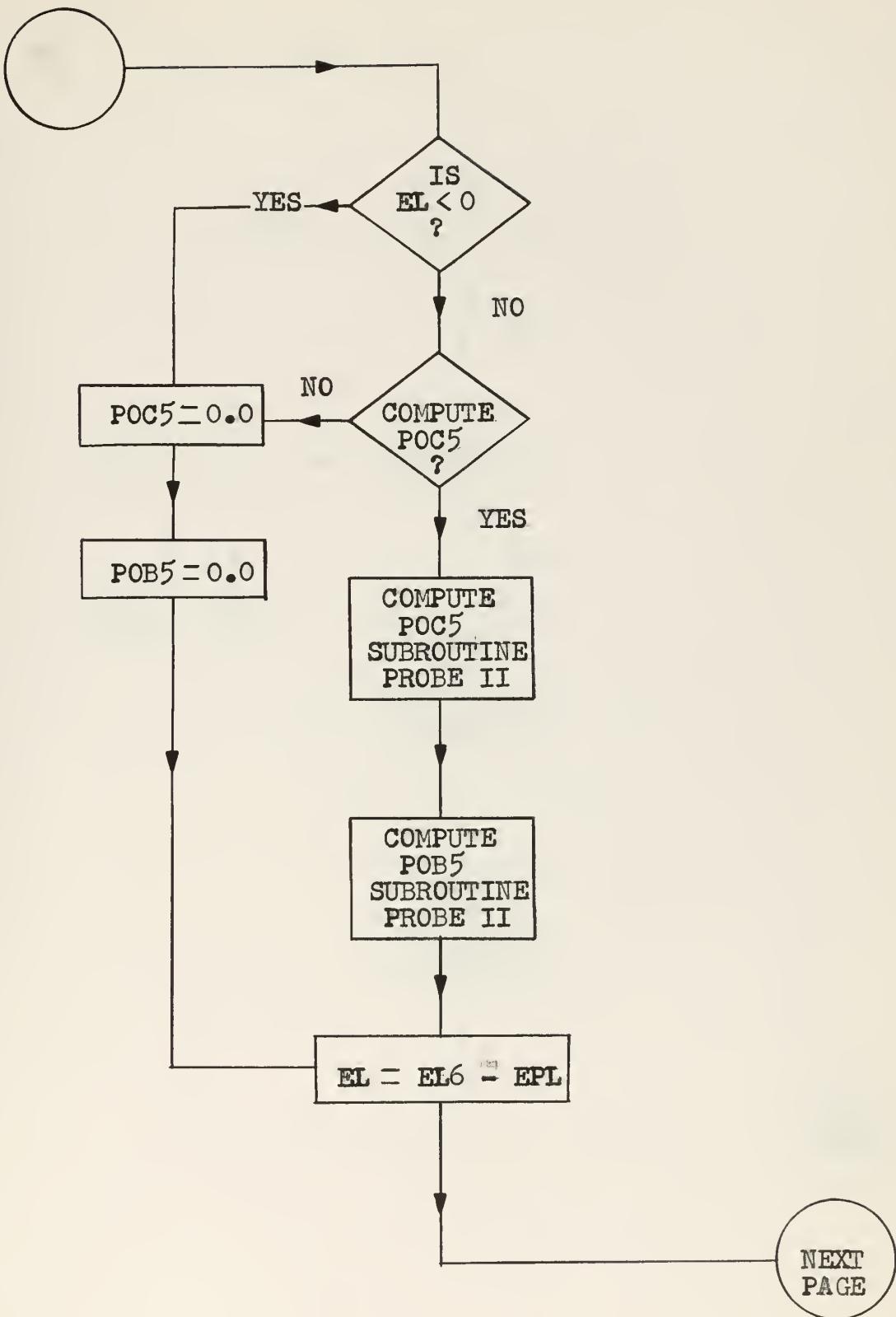


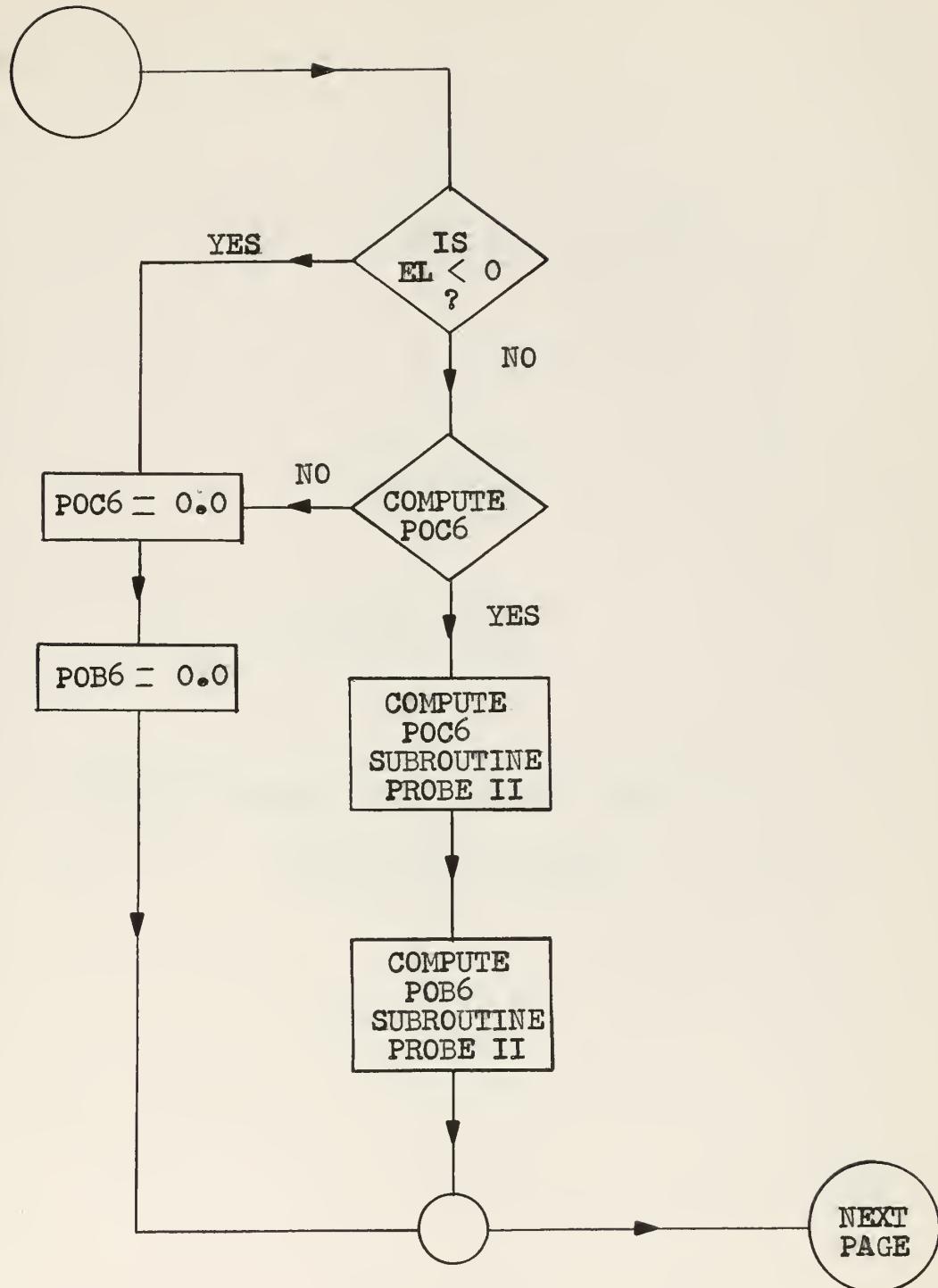


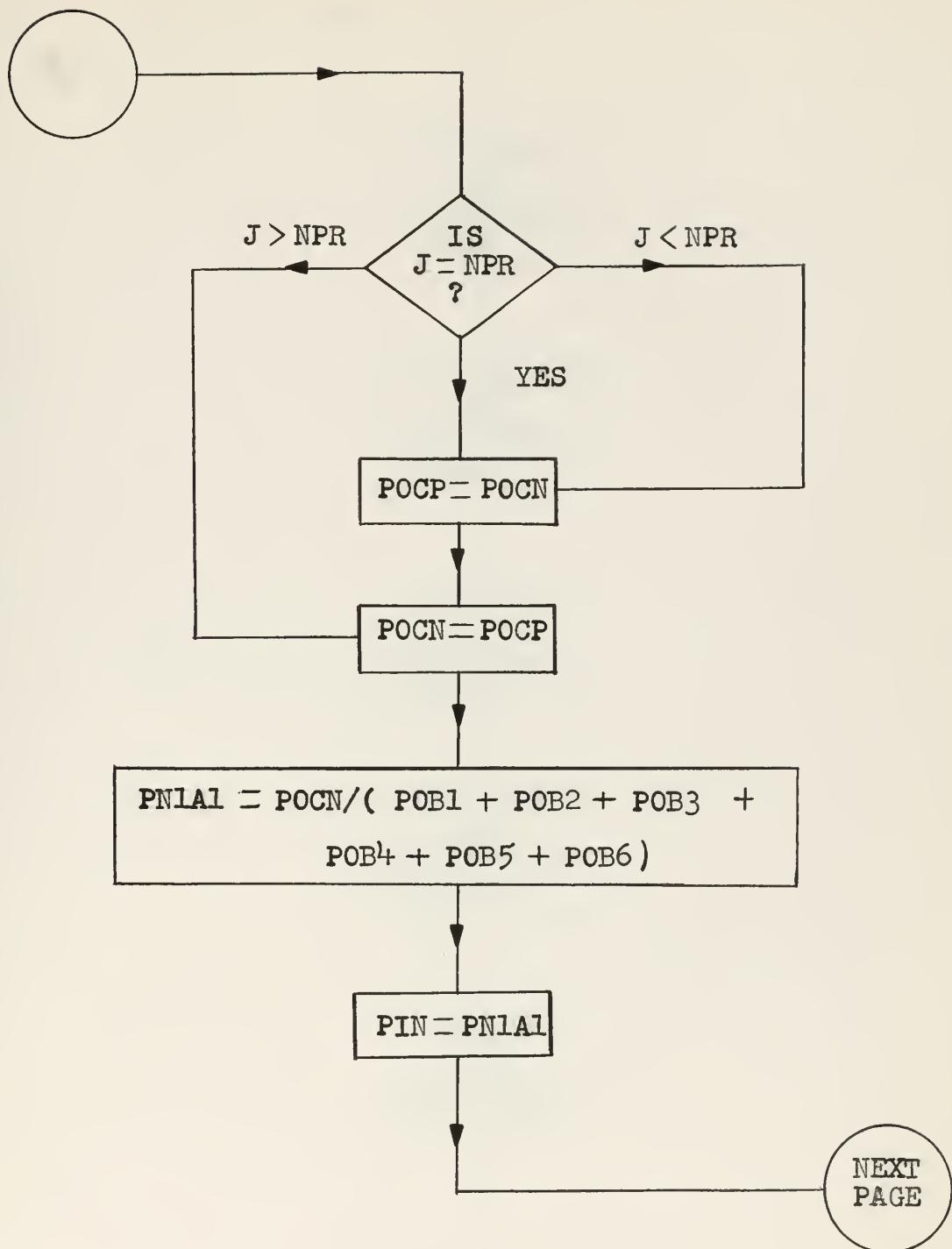


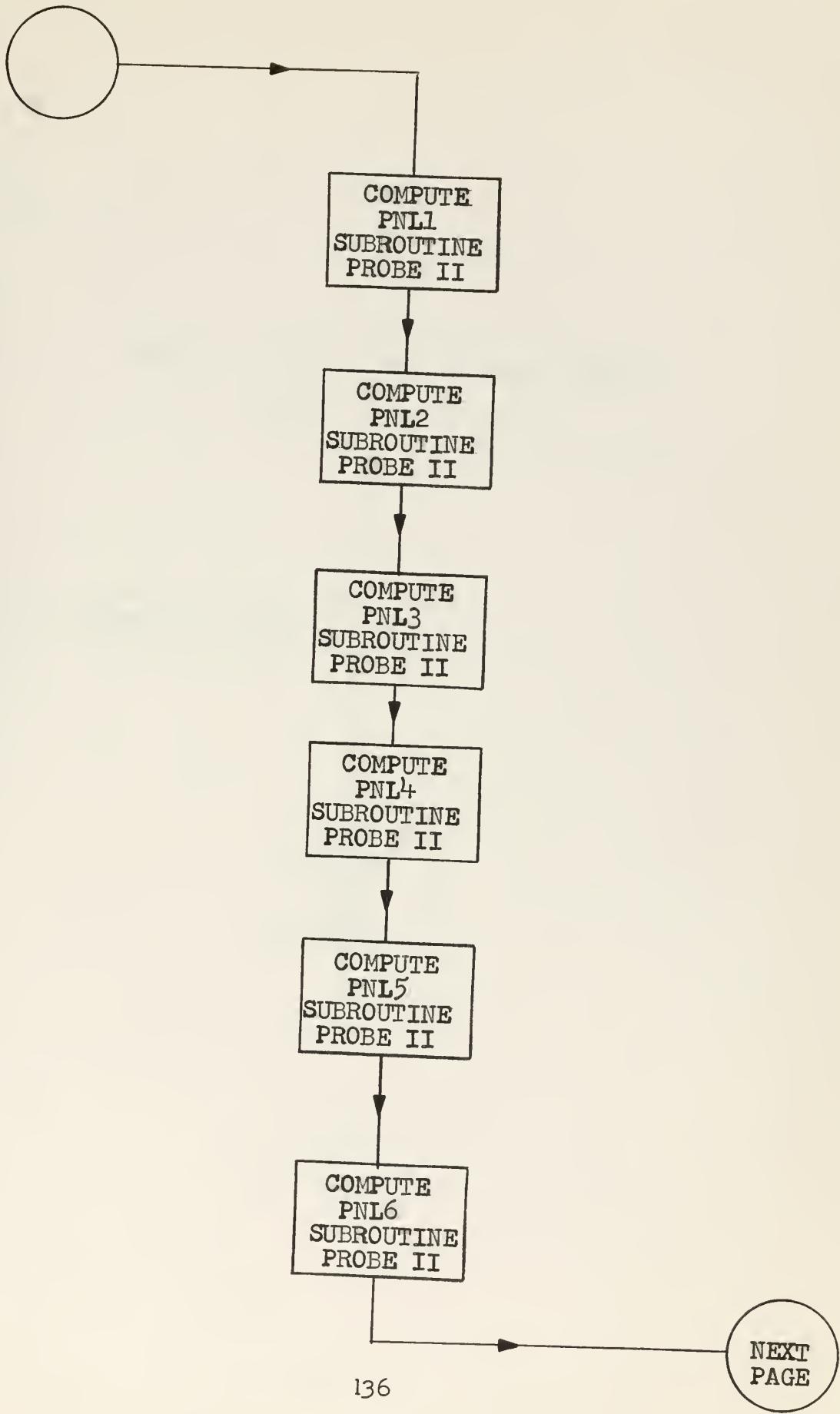


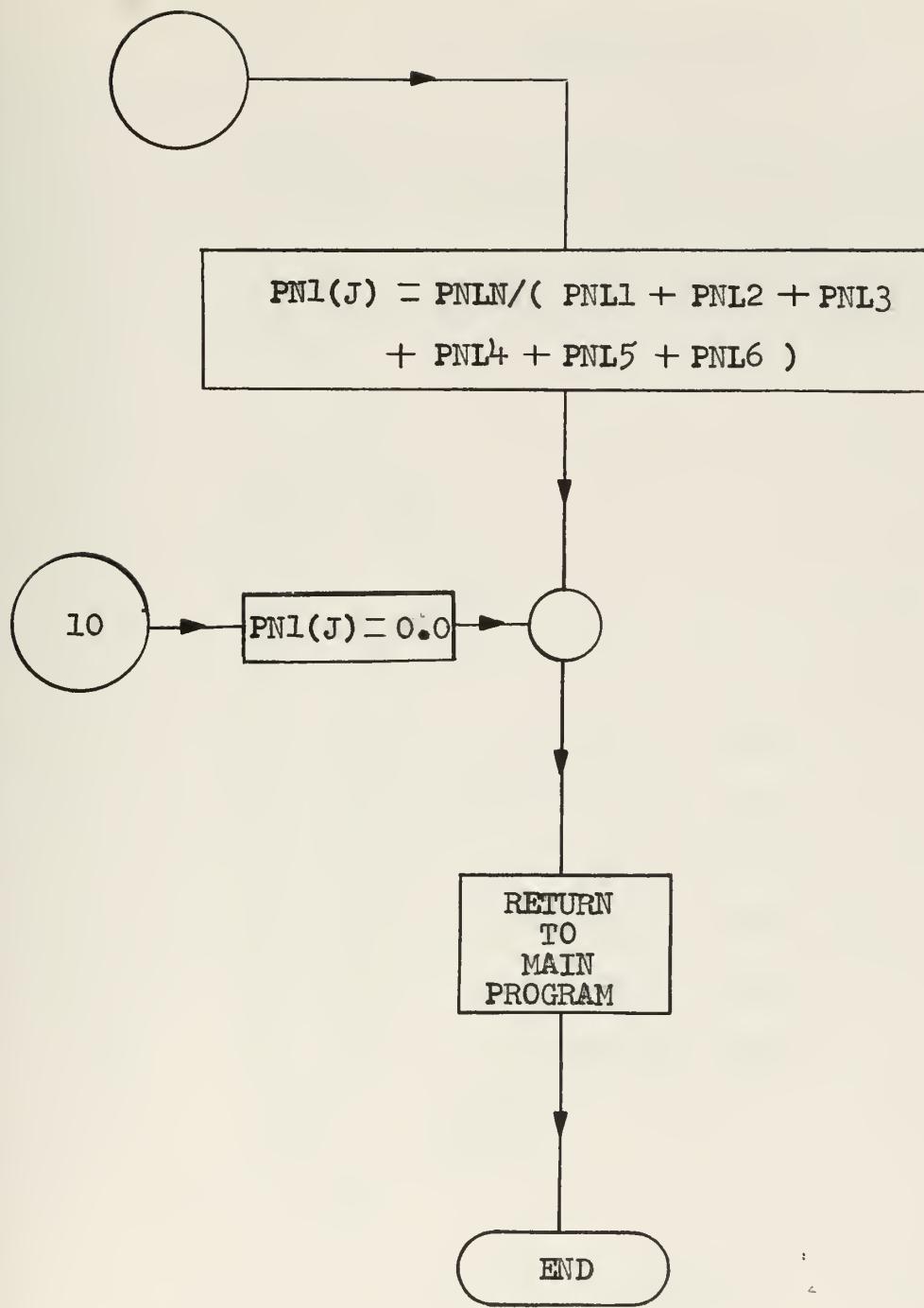












APPENDIX IIB

PROGRAM LISTING

PROGRAM PRODUCE

```

DIMENSION SEA(101),SEC(101),Y(102),ITITLE(12),
1 E(102),PNL1(101),PNL2(101),PNL3(101),PNL4(101),
2 PNC1(101),PNC2(101),PNC3(101),PNC4(101),
3 PNC5(101),PNC6(101),PN7(101),PN6(101),CRN(101),
4 PNR(101),PNK(101),PNP(101),DENOM(101)
COMMON E,POC1,POC2,POC3,POC4,POC5,POC6,
1 PNL1,PNL2,PNL3,PNL4,PNL5,PNL6,RN,C,A,
2 DENOM
RN = 1.0
C = 1.0
A = 1.0
READ 99 SEA
READ 99 SEC
99 FORMAT ( 20F4.0)
E(1) = 0.0
DO 50 J = 1,101
50 E(J + 1) = E(J) + .3
NWB = 101
PIN = 1.0
EL = 7.2
CALL PROD ( EL,PIN,PNC1, NWB )
EL = 9.9
CALL PROD ( EL,PIN,PNC2, NWB )
EL = 17.1
CALL PROD ( EL,PIN,PNC3, NWB )
EL = 17.7
CALL PROD ( EL,PIN,PNC4, NWB )
EL = 23.7
CALL PROD ( EL,PIN,PNC5, NWB )
EL = 26.7
CALL PROD ( EL,PIN,PNC6, NWB )
DO 6161 J = 1,101
PNC22(J) = PNC2(J)
PNC33(J) = PNC3(J)
PNC44(J) = PNC4(J)
6161 CONTINUE
DO 990 J = 1,101
DENOM(J) = PNC1(J) + PNC2(J) + PNC3(J) +
1 PNC4(J) + PNC5(J) + PNC6(J)
IF( J - 57 )52,49,51
49 I = J
51 PNCX = PNC22(I)
GO TO 57
52 PNCX = PNC2(J)

```



```

57 PN6(J) = PNCX/DENOM(J)
  IF( J - 79 )59,53,54
53 K = J
54 PNCY = PNC33(K)
  GO TO 67
59 PNCY = PNC3(J)
67 PN7(J) = PNCY/DENOM(J)
  IF( J - 79 )103,101,102
101 M= J
102 PNCZ = PNC44(M)
  GO TO 77
103 PNCZ = PNC4(J)
77 PNU(J) = PNCZ/DENOM(J)
990 CONTINUE
  CALL GRASP ( 7.2,9.9,17.1,17.7,23.7,26.7,
1 PNP,POC3,PNL2,17.1,9.9,79,
2 2,2,1,2,1,1 )
  CALL GRASP ( 7.2,9.9,17.1,17.7,23.7,26.7,
1 PNR,POC3,PNL1,17.1,7.2,79,
2 2,2,1,2,1,1 )
  CALL GRASP ( 7.2,9.9,17.1,17.7,23.7,26.7,
1 PNU,POC4,PNL2,17.7,9.9,79,
2 2,2,1,1,1,1 )
  DO 1000 J = 1,101
  PNJ(J) = PN4(J) + PNU(J)
  PNK(J) = PN6(J) + PN7(J) + PNR(J) + PNP(J)
  CRN(J) = SEA(J) *(PNJ(J))/ PNK(J)
1000 CONTINUE
  NUMPTS = .101
  MODCURV = 1
  Y(1) = 100.0
  DO 41 J = 2,101
41 Y(J) = CRN(J)
  ITYPE = 0
  LABEL = 4H
  ITITLE(1) = 8HLOWRY
  ITITLE(2) = 8HA = 1.0
  ITITLE(3) = 8HRN= 1.
  ITITLE(4) = 8HE=ENERGY
  ITITLE(5) = 8H1 UNIT
  ITITLE(6) = 8H= 1 MEV
  ITITLE(7) = 8HY=CROSS
  ITITLE(8) = 8HSECTION
  ITITLE(9) = 8H1 UNIT
  ITITLE(10) = 8H= 10 MB
  ITITLE(11) = 8H F - 1
  ITITLE(12) = 8H CU 65
  CALL DRAW ( NUMPTS,E,Y,MODCURV,ITYPE,LABEL,ITITLE,
1 EXSCALE, YSCALE, IXUP, IRIGHT, MODEXAX, MODEYAX, IWIDE,
2 IHIGH, IGRID, LAST )
  MODCURV = 3
  DO 42 J = 2,101
42 Y(J) = SEC(J)
  CALL DRAW ( NUMPTS,E,Y,MODCURV,ITYPE,LABEL,ITITLE,

```



```

1 EXSCALE, YSCALE, IXUP, IRIGHT, MODEXAX, MODEYAX, IWIDE,
2 IHIGH, IGRID, LAST )
END
SUBROUTINE GRASP ( EL1,EL2,EL3,
1 EL4,EL5,EL6,PN20,POCN,PNLN,EPU,EPL,NPT,
2 NP1,NP2,NP3,NP4,NP5,NP6 )
DIMENSION E(102),PN20(101),DENOM(101)
COMMON E,POC1,POC2,POC3,POC4,POC5,POC6,
1 PNL1,PNL2,PNL3,PNL4,PNL5,PNL6,RN,C,A,
2 DENOM
DO 889 J = 1,101
IF(E(J) - EPL )10,3,3
3 CONTINUE
NWB = 10.*((E(J) - EPL)/3.) + 1.0
EP = E(J) - EPU
IF(EP)11,4,4
4 CONTINUE
PIN = 1.0
1 EL = EL1 - EPL
IF(EL)22,12,12
12 GO TO ( 121,22 ),NP1
121 CALL PROBE II ( EL,PIN,POC1,NWB )
GO TO 2
22 POC1 = 0.0
2 EL = EL2 - EPL
IF(EL)28,13,13
13 GO TO ( 131,28 ),NP2
131 CALL PROBE II ( EL,PIN,POC2,NWB )
GO TO 8
28 POC2 = 0.0
8 EL = EL3 - EPL
IF(EL)25,14,14
14 GO TO ( 141,25 ),NP3
141 CALL PROBE II ( EL,PIN,POC3,NWB )
GO TO 5
25 POC3 = 0.0
5 EL = EL4 - EPL
IF(EL)26,15,15
15 GO TO ( 151,26 ),NP4
151 CALL PROBE II ( EL,PIN,POC4,NWB )
GO TO 6
26 POC4 = 0.0
6 EL = EL5 - EPL
IF(EL)27,16,16
16 GO TO ( 161,27 ),NP5
161 CALL PROBE II ( EL,PIN,POC5,NWB )
GO TO 7
27 POC5 = 0.0
7 EL = EL6 - EPL
IF(EL) 29,17,17
17 GO TO ( 171,29 ) ,NP6
171 CALL PROBE II ( EL,PIN,POC6,NWB )
GO TO 30
29 POC6 = 0.0

```



```

30 CONTINUE
GO TO 779
11 PNA = 0.0
GO TO 882
779 CONTINUE
IF( J - NPT ) 800,201,202
201 POCP = POCN
202 POCN = POCP
800 PNA = POCL/ ( POC1 + POC2 + POC3 + POC4 +
1 POC5 + POC6 )
882 CONTINUE
EL = EL1
CALL PROBE II ( EL,PNA,PNL1,J )
EL = EL2
CALL PROBE II ( EL,PNA,PNL2,J )
EL = EL3
CALL PROBE II ( EL,PNA,PNL3,J )
EL = EL4
CALL PROBE II ( EL,PNA,PNL4,J )
EL = EL5
CALL PROBE II ( EL,PNA,PNL5,J )
EL = EL6
CALL PROBE II ( EL,PNA,PNL6,J )
PN20(J) = PNLN/DENOM(J)
GO TO 881
10 PN20(J) = 0.0
881 CONTINUE
POC1 = 0.0
POC2 = 0.0
POC3 = 0.0
POC4 = 0.0
POC5 = 0.0
POC6 = 0.0
PNL1 = 0.0
PNL2 = 0.0
PNL3 = 0.0
PNL4 = 0.0
PNL5 = 0.0
PNL6 = 0.0
889 CONTINUE
RETURN
END
SUBROUTINE PROD ( EL,PIN,PNO,NWB )
DIMENSION E(102),P(101),PNO(101),DENOM(101)
COMMON E, POC1,POC2,POC3,POC4,POC5,POC6,
1 PNL1,PNL2,PNL3,PNL4,PNL5,PNL6,RN,C,A,
2 DENOM
DO 2 K =1,NWB
Z = E(K) - EL
IF(Z)10,3,3
3 H = 0.3
W = 10.0*(Z/3.)
6 NW = W + 1.0
Y = 0.0

```



```

DO 206 L =1,NW
P(L) = C*(Z - Y)*PIN*EXP(A*(Y**RN - Z**RN ))
206 Y = Y + H
SUM I = 0.0
NY = W
DO 207 L=2, NY
207 SUM I = SUM I + P(L)
PNO(K) = H/2.*(P(1) + 2.*SUM I + P(NW))
GO TO 2
10 PNO(K) = 0.0
2 CONTINUE
RETURN
END
SUBROUTINE PROBE II ( EL,PIN,PNO,NWB )
DIMENSION E(102),P(101),DENOM(101)
COMMON E,POC1,POC2,POC3,POC4,POC5,POC6,
1 PNL1,PNL2,PNL3,PNL4,PNL5,PNL6,RN,C,A,
2 DENOM
Z = E(NWB) - EL
IF(Z)10,3,3
3 H = 0.3
W = 10.0*(Z/3.)
6 NW = W + 1.0
Y = 0.0
DO 206 L = 1,NW
P(L) = C*(Z - Y)*PIN*EXP(A*(Y**RN - Z**RN ))
206 Y = Y + H
SUM I = 0.0
NY = W
DO 207 L = 2,NY
207 SUM I = SUM I + P(L)
PNO = H/2.*(P(1) + 2.*SUM I + P(NW))
GO TO 2
10 PNO = 0.0
2 CONTINUE
RETURN
END
END

```


APPENDIX IIC

SUBROUTINE GROPE

TRIPLE INTEGRALS

```

SUBROUTINE GROPE ( EL1,EL2,EL3,EL4,EL5,EL6,
1 PN1,PN3AN,POCN,PNLN,EPU,EPL,EPU1,EPL1,
2 NL1,NL2,NL3,NL4,NL5,NL6,NPQ,NPR,
3 NP1,NP2,NP3,NP4,NP5,NP6 )
      DIMENSION E(102),PN1(101),DENOM(101)
      COMMON E,POC1,POC2,POC3,POC4,POC5,POC6,
1 PNL1,PNL2,PNL3,PNL4,PNL5,PNL6,RN,C,A,
2 DENOM
      DO 999 J = 1, 101
      IF(E(J) - EPL)10,3,3
3 CONTINUE
      NWG = 10.*((E(J) - EPL)/3.) + 1.0
      EP = E(J) - EPL
      IF(E(J) - EPU)11,4,4
4 CONTINUE
      NWB = 10.0*(E(J) - EPL1)/3. + 1.0
      IF(E(J) - EPU1)12,5,5
5 CONTINUE
      PIN = 1.0
21 EL = EL1 - EPL1
      IF(EL)72,14,14
14 GO TO ( 141,72 ),NL1
141 CALL PROBE II ( EL,PIN,PN3A1,NWB )
      GO TO 22
72 PN3A1 = 0.0
22 EL = EL2 - EPL1
      IF(EL)73,15,15
15 GO TO ( 151,73 ),NL2
151 CALL PROBE II ( EL,PIN,PN3A2,NWB )
      GO TO 23
73 PN3A2 = 0.0
23 EL = EL3 - EPL1
      IF(EL)74,16,16
16 GO TO ( 161,74 ),NL3
161 CALL PROBE II ( EL,PIN,PN3A3,NWB )
      GO TO 24
74 PN3A3 = 0.0
24 EL = EL4 - EPL1
      IF(EL)75,17,17
17 GO TO ( 171,75 ),NL4
171 CALL PROBE II ( EL,PIN,PN3A4,NWB )
      GO TO 25
75 PN3A4 = 0.0
25 EL = EL5 - EPL1

```



```

    IF(EL)76,18,18
18 GO TO ( 181,76 ),NL5
181 CALL PROBE II ( EL,PIN,PN3A5,NWB )
    GO TO 26
26 PN3A5 = 0.0
    IF(EL) 77,19,19
19 GO TO (191,77 ),NL6
191 CALL PROBE II ( EL,PIN,PN3A6,NWB )
    GO TO 78
78 PN3A6 = 0.0
79 CONTINUE
    GO TO 993
12 PN3A = 0.0
    GO TO 995
    IF( J - NPQ ) 993,201,202
201 PN3AP = PN3AN
202 PN3AN = PN3AP
993 CONTINUE
    PN3A = PN3AN/( PN3A1 + PN3A2 + PN3A3 + PN3A4 +
1 PN3A5 + PN3A6 )
995 CONTINUE
31 EL = EL1 - EPL
    IF(EL)172,42,42
42 GO TO ( 421,172 ),NP1
421 CALL PROBE II ( EL,PN3A,POC1,NWC )
    CALL PROBE II (EL,1.0,POB1,NWC )
    GO TO 32
172 POC1 = 0.0
    POB1 = 0.0
32 EL = EL2 - EPL
    IF(EL)173,43,43
43 GO TO ( 431,173 ),NP2
431 CALL PROBE II ( EL,PN3A,POC2,NWC )
    CALL PROBE II (EL,1.0,POB2,NWC )
    GO TO 33
173 POC2 = 0.0
    POB2 = 0.0
33 EL = EL3 - EPL
    IF(EL)174,44,44
44 GO TO ( 441,174 ),NP3
441 CALL PROBE II ( EL,PN3A,POC3,NWC )
    CALL PROBE II (EL,1.0,POB3,NWC )
    GO TO 34
174 POC3 = 0.0
    POB3 = 0.0
34 EL = EL4 - EPL
    IF(EL)175,45,45
45 GO TO ( 451,175 ),NP4
451 CALL PROBE II ( EL,PN3A,POC4,NWC )
    CALL PROBE II (EL,1.0,POB4,NWC )
    GO TO 35
175 POC4 = 0.0
    POB4 = 0.0

```



```

35 EL = EL5 - EPL
    IF(EL)176,46,46
46 GO TO ( 461,176 ),NP5
461 CALL PROBE II ( EL,PN3A,POC5,NWC )
    CALL PROBE II ( EL,1.0,POB5,NWC )
    GO TO 36
176 POC5 = 0.0
    POB5 = 0.0
36 EL = EL6 - EPL
    IF(EL)177,47,47
47 GO TO ( 471,177 ),NP6
471 CALL PROBE II ( EL,PN3A,POC6,NWC )
    CALL PROBE II ( EL,1.0,POB6,NWC )
    GO TO 48
177 POC6 = 0.0
    POB6 = 0.0
48 CONTINUE
    GO TO 992
11 PN1A1 = 0.0
    GO TO 994
    IF ( J - NPR ) 992,901,902
901 POCP = POCN
902 POCN = POCP
992 CONTINUE
    PN1A1 = POCN/( POB1 + POB2 + POB3 +
    1 POB4 + POB5 + POB6 )
994 CONTINUE
    EL = EL1
    CALL PROBE II ( EL,PN1A1,PNL1,J )
    EL = EL2
    CALL PROBE II ( EL,PN1A1,PNL2,J )
    EL = EL3
    CALL PROBE II ( EL,PN1A1,PNL3,J )
    EL = EL4
    CALL PROBE II ( EL,PN1A1,PNL4,J )
    EL = EL5
    CALL PROBE II ( EL,PN1A1,PNL5,J )
    EL = EL6
    CALL PROBE II ( EL,PN1A1,PNL6,J )
    PN1(J) = PNLN/DENOM(J)
    GO TO 991
10 PN1(J) = 0.0
991 CONTINUE
    POC1 = 0.0
    POC2 = 0.0
    POC3 = 0.0
    POC4 = 0.0
    POC5 = 0.0
    POC6 = 0.0
    PNL1 = 0.0
    PNL2 = 0.0
    PNL3 = 0.0
    PNL4 = 0.0
    PNL5 = 0.0

```


PNL6 = 0.0
PN3A1 = 0.0
PN3A2 = 0.0
PN3A3 = 0.0
PN3A4 = 0.0
PN3A5 = 0.0
PN3A6 = 0.0
POB1 = 0.0
POB2 = 0.0
POB3 = 0.0
POB4 = 0.0
POB5 = 0.0
POB6 = 0.0
999 CONTINUE
RETURN
END

APPENDIX III

DENSITY FUNCTIONS

1. $P(E) = C(Z - E) \exp(-A(E^n - Z^n)) ; Z = E - E_{th}$
2. $P(E) = C \exp(-AE^n)$
3. $P(E) = C$
4. $P(E) = C Z \exp(-A(E^n - E_{th}^n))$
5. $P(E) = C Z \exp(-AE^n)$
6. $P(E) = C Z \exp(-A(E^n - Z^n))$
7. $P(E) = C E \exp(-AE^n)$
8. $P(E) = C E \exp(-A(E^n - Z^n))$
9. $P(E) = C E \exp(-A(E^n - E_{th}^n))$
10. $P(E) = C/\exp(-AE^n)$
11. $P(E) = C E \exp(-Z(AE)^n)$
12. $P(E) = C E \exp(-Z AE)$
13. $P(E) = C E/\exp(-AE^n)$
14. $P(E) = C(E)^3 \exp(-AE^n)$

APPENDIX IV

TEST

In order to test the program, it was postulated that if all threshold values were set equal to the same value, the output of the program would be constant and the results would be the input multiplied by that constant.

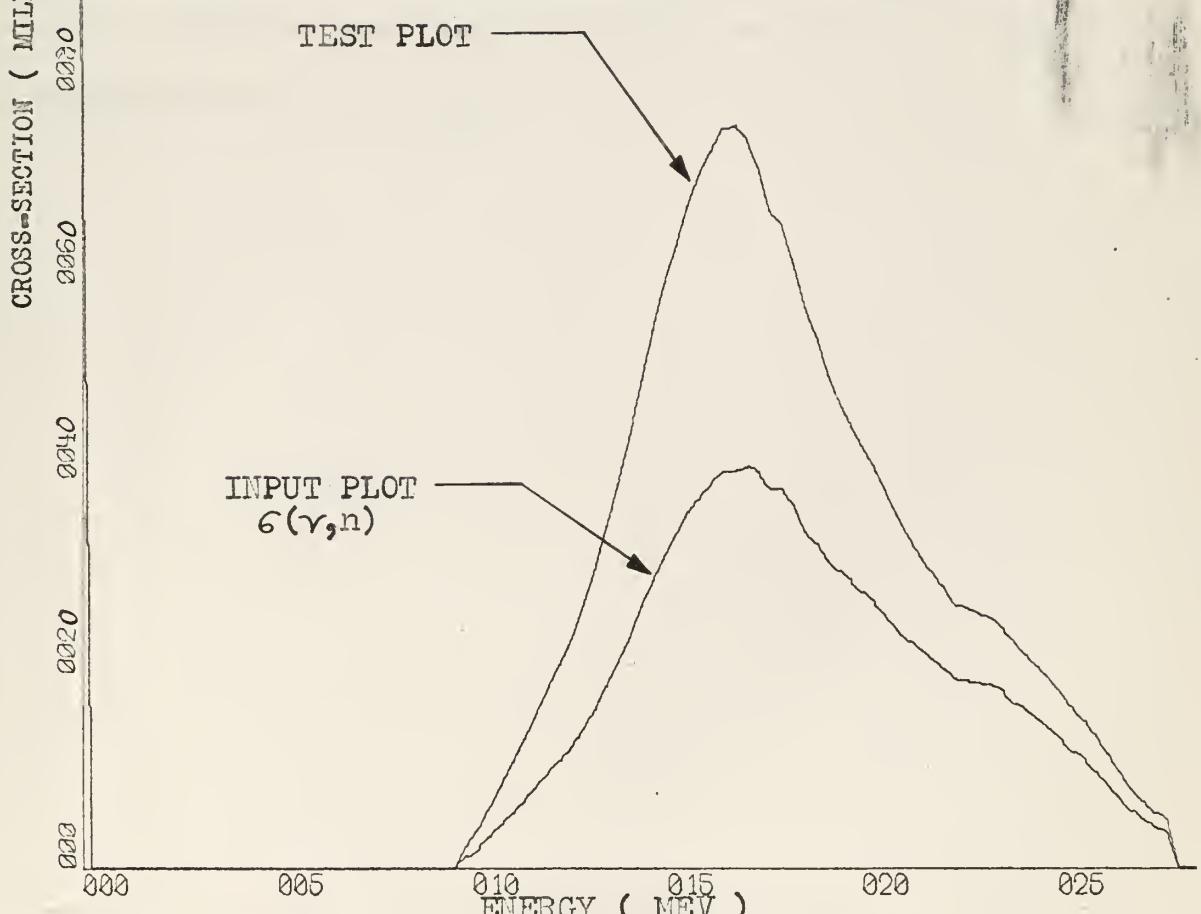
In carrying out this test, the density function:

$$P(E) = C E \exp(-A E^n)$$

was used. All threshold values were set equal to 7.2 MEV, the lowest threshold value.

The result is a graph which is the plot of the input multiplied by a constant as expected. See Figure 65.

CROSS-SECTION
vs
ENERGY
of
INCIDENT GAMMA-RAY



DENSITY FUNCTION $P(E) = C E \exp(-\Lambda E^n)$

$$\Lambda = 0.001 \text{ MEV}^{-1}$$

TEST

FIGURE 65

Acknowledgements

I am grateful to Dr. Fred R. Buskirk for his untiring advice and leadership in directing me through this project. His knowledge and experience in theoretical physics have been a continuing source of enlightenment.

I would also like to thank the personnel in the computer center. In particular, I wish to thank the operators who ran my multitude of programs through "the machine", and the professional programmers who helped me find the "bugs".

thesL87

Statistical model calculations for photo



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